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ELEMENTS OF THE MATHEMATICAL THEORY OF FLUID MOTION.

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II.

§ 2.

THE POTENTIAL.

It will perhaps be as well to make a few remarks here concerning the theory of the potential. It is not the purpose in these pages to go into that subject with any degree of fullness, but as there are a few leading principles which frequently recur a brief statement and derivation of them may be of assistance to some readers.

We have already observed one fact concerning the velocity potential, viz., that if it is constant over a closed surface containing a certain definite region that it will be constant throughout this region, and in particular, if it be $=o$ over the surface it will be $=o$ throughout the contained region. When $udx+vdy+wdz$ is an exact differential we have seen that by making it equal to $d\varphi$ we can replace the quantity

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}$$

by

$$\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz^2}.$$

Now let σ denote any closed surface, then if ν be the outer normal to this surface we have $\frac{d\varphi}{d\nu}\,d\sigma$ for the rate of flow outwards through the element $d\sigma$ in unit of time, then the total flow outwards in time dt is equal to

$$\int\int\frac{d\varphi}{d\nu}d\sigma.dt$$

where the integration extends over the whole surface. If the space enclosed by σ be full both at the beginning and end of this time we, of course, have

$$\int\int\frac{d\varphi}{d\nu}d\sigma=o.$$

This is the equation of continuity for the whole region. Applied to the element of volume $d\tau$ or $dx\,dy\,dz$ this gives us

$$\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz^2}=o \text{ or } \Delta^2\varphi=o$$

If we denote by r the distance between any two points x, y, z, and a, b, c, *i. e.*

$$r^2=(x-a)^2+(y-b)^2+(z-c)^2$$

and by m a constant we see that the equation $\Delta^2\varphi=o$ is satisfied by

$$\varphi=\frac{m}{r}$$

or $\frac{m}{r}$ is a particular solution of the partial differential equation.

The general solution is a homogeneous function of the quantities x, y, z of the degree i, where i is any positive integer. It is also well known that to every solution of the degree i then corresponds one of the degree $-(i+1)$ expressed by $\frac{\varphi^i}{r^{2i+1}}$.

The expression φ_i is a Solid Spherical Harmonic of the degree i. The expression obtained by dividing φ^i by r^i which will be a function only of two quantities, viz., the angles θ and ψ, is a Spherical Surface Harmonic of the same degree.

The quantity φ is now the Potential of the mass m upon the point (x, y, z). At infinity the Potential with its derivatives vanishes, but is finite and continuous throughout the space except at the points in which the masses are found, *i. e.*, for $x=a$, $y=b$, $z=c$. Let now in the space under consideration, which is supposed to be continuously filled with masses, m denote a mass placed at a given point A and let r denote the length of a line drawn from this point to any other B—then we know that the attraction of the mass m upon the point B is given by $-\frac{d\varphi}{dr}=\frac{m}{r^2}$.

Now suppose the line AB drawn to an infinite distance, and further that the space which contains the masses m is bounded by a closed surface. The line AB will cut the surface an even number of times. Suppose now a sphere of radius unity to be described with center A, and then let the line AB describe a conical surface cutting the element $d\omega$ from the surface of the sphere, and ds_1, ds_2, &c., from the given closed surface. Let ε denote the angle between AB and the outer normal to the surface; then where the line issues from the surface cos. ε will be positive and where it enters cos. ε will be negative. We have now

$$ds=\frac{r^2 d\omega}{\cos.\ \varepsilon}$$

The normal force on the element ds is given by R cos. $\varepsilon\ ds$; but R cos. $\varepsilon\ ds$ is equal to

$$\left(\frac{dX}{dx}+\frac{dY}{dy}+\frac{dZ}{dz}\right)dx\,dy\,dz$$

and therefore for the whole surface

$$\int\int \mathrm{R}\cos.\ \varepsilon\ ds=\int\int\int\left(\frac{dX}{dx}+\frac{dY}{dy}+\frac{dZ}{dz}\right)dx\,dy\,dz$$

Now since $\mathrm{R}=\frac{m}{r^2}$ we have

$$\mathrm{R}\cos.\ \varepsilon\ ds=\pm\ m d\omega.$$

As the point A is within the surface, the line AB first issues from the surface, giving a positive value of $md\omega$ after that alternate positive and negative values of $md\omega$ which destroy each other so that we have simply

$$\Sigma\ \mathrm{R}\cos.\ \varepsilon\ d\omega=md\omega$$

and

$$\int\int \mathrm{R}\cos.\ \varepsilon\ d\omega=m\int\int d\omega=4\pi m$$

Now

$$4\pi m=4\pi\int\int\int\rho\ dx\,dy\,dz$$

$$\therefore\ \frac{dX}{dx}+\frac{dY}{dy}+\frac{dZ}{dz}=4\pi\rho$$

or

$$\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz^2}=-4\pi\rho \text{ for } \mathrm{X}=-\frac{d\varphi}{dx}\ \&c.$$

This equation holds throughout the entire space which is filled with the masses m.

If we represent an element of the space under consideration by $d\tau$ and the density by ρ we may write

$$\varphi=\int\frac{\rho d\tau}{r}$$

from which

$$\frac{d\Phi}{dz}=\int\rho\frac{d\frac{1}{r}}{dz}d\tau=-\int\rho\frac{d\frac{1}{r}}{dc}d\tau$$

$$=-\int\frac{d\frac{\rho}{r}}{dc}d\tau+\int\frac{d\rho}{dc}\frac{d\tau}{r}$$

and finally

$$\frac{d\varphi}{dz}=\int ds\frac{\rho}{r}\cos.\left(\frac{n}{z}\right)+\int\frac{d\rho}{dc}\frac{d\tau}{d}$$

The second of these integrals is evidently a quantity of the same kind as φ; the first is the potential of a mass which is spread out upon the surface $\int ds$ and giving the surface density

$$\rho\cos.\left(\frac{n}{z}\right).$$

Suppose now that φ' be the potential with reference to the point (x, y, z) of a mass that is spread out upon a surface giving the surface density μ; we have

$$\varphi' = \int \frac{\mu d\sigma}{\sigma}.$$

Assume the rectangular axes so that z is normal to the surface; assume on d a point infinitely near the surface, and suppose a circular cylinder with radius P′ and having the axis of z for its axis of figure to cut the surface; suppose P′ indefinitely small but infinitely large with respect to the ordinate z. Let φ_1' denote the portion of φ belonging to that part of surface included in the cylinder; the remaining portion $\varphi' - \varphi_1'$ will not become infinite or discontinuous by z becoming either $=o$ or passing through zero.

$$\varphi_1' = 2\pi\mu \int_o^{P'} \frac{\rho' d\rho'}{\sqrt{\rho'^2 + z^2}}$$

$$\varphi_1' = 2\pi\mu [\sqrt{P'^2 + z^2} - \sqrt{z^2}]$$

By neglecting infinitesimals we have

$$\varphi_1' = o$$

or φ' remains finite and continuous if the point under consideration passes through the surface, *i.e.*, if $\sqrt{z^2} = \pm z$. Further.

$$\frac{d\varphi_1'}{dz} = 3\pi\mu \left\{ \frac{z}{\sqrt{P'^2 + z^2}} - \frac{z}{\sqrt{z^2}} \right\}$$

or since with respect to z P′ is infinitely great,

$$\frac{d\varphi_1'}{dz} = -2\pi\mu \frac{z}{\sqrt{z^2}}$$

or if z be positive

$$\frac{d\varphi_1'}{dz} = -2\pi\mu$$

if z be negative

$$\frac{d\varphi_1'}{dz} = +2\pi\mu.$$

But $\varphi' - \varphi_1'$ is continuous and also $\frac{d(\varphi' - \varphi_1')}{dz}$; consequently when z changes from positive to negative passing through zero $\frac{d\varphi'}{dz}$ changes suddenly by the amount $-4\pi\mu$. Now as we have taken z in the direction of the normal this fact can be expressed as follows: Call the inner normal ν_1 and the outer normal ν_2 then we have

$$\frac{d\varphi'}{d\nu_1} + \frac{d\varphi'}{d\nu_2} = -4\pi\mu$$

which may be called the *characteristic equation* of φ' at the surface.*

Suppose now that we have a surface over which a mass is so distributed as to give rise to the surface density that we have denoted by μ giving the n the potential φ'. We will use, for the present, the symbol φ to express general function, which satisfies the equation,

$$\frac{d^2\varphi}{dx^2} + \frac{d^2\varphi}{dy^2} + \frac{d^2\varphi}{dz^2} = o.$$

And we will take now U to represent the potential in the space under consideration, V for what we have denoted by φ', the potential of a surface over which a mass is distributed, as mentioned above. Now we will introduce a new quantity W, which we proceed to define: At every point of the surface that we have spoken of conceive normals (positive) to be drawn, lay off on these infinitely small lengths, and through these points conceive another surface to pass, the elements of which correspond to the elements of the first surface and consider that on each element of this new surface a mass is distributed equal to that on the corresponding element of the first surface but of opposite sign. Represent by k the negative product of the density of the mass on the element $d\sigma$ of the first surface, by $d\sigma$; then if (a, b, c) denote the element $d\sigma$ and W is the potential of this element at the point (x, y, z); we have

$$W = \int k \frac{d\frac{1}{r}}{d\nu} d\sigma$$

Since, $\frac{d\frac{1}{r}}{dr} = \frac{d\frac{1}{r}}{dr} \frac{dr}{d\nu}$

we have

$$W = -\int \frac{k d\sigma}{r^2} \cos.\ (r\nu)$$

We can also express this in another way. Conceive a sphere of unit radius described about $(x, y, z,)$ as center, also conceive a cone on $d\sigma$ as base and having (x, y, z) as vertex, to cut the sphere, the area of the included portion being $d\Sigma$, then

* Maxwell, Elec. and Mag., p. 82, Vol. I.

$$\frac{d\frac{1}{r}}{d\nu}d\sigma = \pm d\Sigma$$

the upper or lower sign to be taken according as cos. $(r\nu)$ is positive or negative. The cosine can only change its sign by $(r\nu)$ passing through $\frac{\pi}{2}$ evidently this is the case only when the point can lie on a tangent to the surface; then supposing that cos. $(r\nu)$ does not change its sign, we have

$$W = \pm \int k d\Sigma$$

when the upper or lower sign is to be taken according as cos. $(r\nu)$ is positive or negative. If the above condition is not fulfilled, the surface may be divided into parts so that each part can satisfy the imposed conditions; then will W begin as the sum of the corresponding expressions for each part. In order to examine whether or not discontinuity occurs in the value of W, by the point to which it refers, approaching indefinitely near the surface —coinciding with it or passing through it—we will choose the axis so that z is normal to the surface, and assume a point on z indefinitely near the surface now by exactly the same process as that before employed, we see that for a negative z we have

$$W_1 = -z\pi k$$

and for a positive z

$$W_1 = +z\pi k$$

W_1 corresponding to the small portion cut out of the surface by a circular cylinder of indefinitely small radius P, which is nevertheless infinitely large as regards z. Now since W_1 is independent of z, W cannot become infinitely great by z becoming infinitely small; and since W_1 suddenly changes by $4\pi k$, W does so likewise. From our equation for W, we have for W_1 since it refers to an indefinitely small portion of the surface for which k is constant,

$$W_1 = -k\int \frac{d\frac{1}{r}}{dz} d\sigma = 2\pi k \int_0^P \frac{z\rho d\rho}{(\rho^2+z^2)^{\frac{3}{2}}}$$

since

$$r = \sqrt{\rho^2+z^2} \text{ and } \frac{d\frac{1}{r}}{dz} = \frac{d\frac{1}{r}}{dd}\frac{dr}{dz} = -\frac{1}{r^2}\frac{dr}{dz},$$

or

$$W_1 = z\pi k\left\{\frac{z}{\sqrt{z^2}} - \frac{z}{\sqrt{P^2+z^2}}\right\}$$

from this follows

$$\frac{dW_1}{dz} = -z\pi k\frac{P^2}{(P^2+z^2)^{\frac{3}{2}}}$$

or, since z^2 is negligable with respect to P^2,

$$\frac{dW_1}{dz} = -2\pi k\frac{1}{P}$$

The second member being independent of z, $\frac{dW'}{dz}$ for $z=o$ is finite and continuous. Thus we see that the potential W for this *double layer* is finite always, but changes suddenly by the amount $4\pi k$ on the point to which it refers passing through the surface in the direction of ν; the quantity $\frac{dW}{d\nu}$ is, however, finite and continuous. With a perfectly arbitrary co-ordinate system the quantities $\frac{dW}{dx}$, $\frac{dW}{dy}$, $\frac{dW}{dz}$, will in general suffer discontinuity, since k is in general not constant over all the surface; if k be constant the differential co-efficients will suffer no discontinuity at the surface.

Suppose that we have two functions U and V of x, y, z, which with their derivations are single valued and continuous in the space under consideration, which is bounded by a closed surface. We have the identical equations

$$\frac{dU}{dx}\frac{dV}{dx} + U\frac{d^2V}{dx^2} = \frac{d}{dx}\left(U\frac{dV}{dx}\right),$$

$$\frac{dU}{dy}\frac{dV}{dy} + U\frac{d^2V}{dy^2} = \frac{d}{dy}\left(U\frac{dV}{dy}\right),$$

$$\frac{dU}{dz}\frac{dV}{dz} + U\frac{d^2V}{dz^2} = \frac{d}{dz}\left(U\frac{dV}{dz}\right).$$

Add and multiply by the element of the space $d\tau$, we have by changing in the second member a volume into a surface integral,

$$(A)\int\left(\frac{dU}{dx}\frac{dV}{dx} + \frac{dU}{dy}\frac{dV}{dy} + \frac{dU}{dz}\frac{dV}{dz}\right)d\tau = -\int d\tau\, U\,\Delta V - \int d\sigma U\frac{dV}{d\nu}.$$

This equation expresses what is known as Green's Theorem. By an interchange of U and V, which from the nature of the functions can be effected, the first member of this equation will not change. We will have then

$$(B)\int d\sigma\left(U\frac{dV}{d\nu}+V\frac{dU}{d\nu}\right)$$
$$=\int d\tau(V\Delta U-U\Delta V)$$

If U and V are velocity potentials this gives,

$$\int d\sigma\left(U\frac{dV}{d\nu}-V\frac{Ud}{d\nu}\right)=o$$

In the preceding equation (a) suppose U=V, then it gives,

$$\int d\tau\left\{\left(\frac{dV}{dx}\right)^2+\left(\frac{dV}{dy}\right)^2+\left(\frac{dV}{dz}\right)^2\right\}$$
$$=-\int d\sigma V\frac{dV}{d\nu}$$

or

$$2T=-\int V\left\{\frac{dV}{dx}\cos.(\nu x)+\frac{dV}{dy}\cos.(\nu y)\right.$$
$$\left.+\frac{dV}{dz}\cos.(\nu z)\right\}d\sigma$$

An expression for the energy in the form of a surface integral.

"The irrotational motion of incompressible fluid in a simply connected closed space Σ is completely determined by the normal velocities over the surface Σ. If Σ be a material envelope, it is evident that an arbitrary normal velocity may be impressed upon its surface, which normal velocity must be shared by the fluid immediately in contact, provided that the whole volume inclosed remain unaltered. If the fluid be previously at rest, it can acquire no molecular rotation under the operation of the fluid pressures, which shows that it must be possible to determine a function φ, such that $\Delta^2\varphi=o$ throughout the space inclosed by Σ while over the surface $\frac{d\varphi}{d\nu}$ has a prescribed value limited only by the condition

$$\int\int\frac{d\varphi}{d\nu}d\sigma=o$$

"By Green's theorem if $\Delta^2\varphi=o$,

$$\int\int\int\left\{\left(\frac{d\varphi}{dx}\right)^2+\left(\frac{d\varphi}{dy}\right)^2+\left(\frac{d\varphi}{dz}\right)^2\right\}d\tau=$$
$$\int\int\varphi\frac{d\varphi}{d\nu}d\sigma$$

the integration on the right hand side extending over the surface Σ that on the left hand side over the volume. Now if φ and $\varphi+\Delta\varphi$ be two functions, satisfying Laplace's equation, and giving prescribed values of $\frac{d\varphi}{d\nu}$, then the difference, $\Delta\varphi$ is a function also satisfying Laplace's equation, and making $\frac{\Delta d\varphi}{d\nu}$ vanish over the surface of Σ. Under these circumstances the surface integral in the preceding equation vanishes, and we infer that at every point of Σ, $\frac{d\Delta\varphi}{dx}$, $\frac{d\Delta\varphi}{dy}$, $\frac{d\Delta\varphi}{dz}$ must be equal to zero. In other words, $\Delta\varphi$ must be constant and the two motions identical. As a particular case, there can be no motion of the irrotational kind within the volume Σ, independently of a motion of the surface."

The line described by a point in the fluid, moving always in the direction of the resultant velocity, is, as has been mentioned for a simple case, called a stream line. If φ denote the velocity potential, we have obviously for the differential equations of a stream line,

$$\frac{dx}{\frac{d\varphi}{dx}}=\frac{dy}{\frac{d\varphi}{dy}}=\frac{dz}{\frac{d\varphi}{dz}}$$

These lines evidently cut at right angles the surface φ=const.

If the normal velocity at every point of this surface is equal to zero, we must have

$$\frac{d\varphi}{d\nu}=o$$

and this equation will represent a surface which cuts at right angles the surface φ=const., and is consequently made up of stream lines; such a surface is appropriately termed a surface of flow. The following properties of such surfaces, though not directly bearing upon the matter in hand, will possibly be of interest:

Suppose that we have for the equation of a family of surfaces

$$f(x, y, z)=q,$$

q being a variable parameter by giving constant values to which we obtain the equation of each member of the family. Make

$$\left(\frac{dq}{dx}\right)^2+\left(\frac{dq}{dy}\right)^2+\left(\frac{dq}{dz}\right)^2=\frac{1}{\nu^2}$$

Then we have for the direction cosines of the normal in the direction in which q increases

$$\text{cos. } (\nu x) = \nu \frac{dq}{dx}, \cos(\nu y) = \nu \frac{dq}{dy},$$

$$\text{cos.}(\nu z) = \nu \frac{dq}{dz}$$

if N be the component of the flow normal to the surface, u, v, w being the components parallel to x, y and z respectively, we have

$$\mathrm{N} = \nu \left(u \frac{dq}{dx} + v \frac{dq}{dy} + w \frac{dq}{dz} \right).$$

Hence, if N be zero, there will be no flow through the surface, which may then be called a surface of flow; we have then for the equation of such a surface,

$$u \frac{dq}{dx} + v \frac{dq}{dy} + w \frac{dq}{dz} = o.$$

If there be another family of surfaces whose parameter is q', and these are surfaces of flow, then

$$u \frac{dq'}{dx} + v \frac{dq'}{dy} + w \frac{dq'}{dz} = o$$

If we have still a third family of surfaces whose parameter is q'' that are surfaces of flow, then again,

$$u \frac{dq''}{dx} + v \frac{dq''}{dy} + v \frac{dq''}{dz} = o$$

eliminating u, v and w between these, and we obtain

$$\begin{vmatrix} \frac{dq}{dx}, & \frac{dq}{dy}, & \frac{dq}{dz} \\ \frac{dq'}{dx}, & \frac{dq'}{dy}, & \frac{dq'}{dz} \\ \frac{dq''}{dx}, & \frac{dq''}{dy}, & \frac{dq''}{dz} \end{vmatrix} = o$$

which is only satisfied by making $q'' =$ some function of q and q'. Suppose that we have only the two first of these equations viz.:

$$u \frac{dq}{dx} + v \frac{dq}{dy} + w \frac{dq}{dz} = o$$

$$u \frac{dq'}{dx} + v \frac{dq'}{dy} + w \frac{dq'}{dz} = o$$

By eliminating u, v, w in turn from these equations we arrive at the following where Φ is an undetermined function of q and q',

$$u = \Phi \left(\frac{dq}{dy} \frac{dq'}{dz} - \frac{dq}{dz} \frac{dq}{dy} \right)$$

$$v = \Phi \left(\frac{dq}{dz} \frac{dq'}{dx} - \frac{dq}{dx} \frac{dq'}{dz} \right)$$

$$w = \Phi \left(\frac{dq}{dx} \frac{dq'}{dy} - \frac{dq}{dy} \frac{dq'}{dx} \right)$$

When one of the functions represented by q and q' is known, it is possible so to determine the other, that Φ shall be $=$ unity. The flow in the direction of the normals to these surfaces being $= o$, this flow can only take place along the surface, and the intersection of the two surfaces will be a line of flow or stream line. A tube of flow or a stream filament, is a tube whose bounding surfaces are made up of lines of flow. If the two parameters q and q' have a series of values given to them, they will form a double system of surfaces dividing space up into a number of tubes, each of which will be a tube of flow. We will go further into the consideration of stream lines in another place.

§ 3.

PLANE WAVES.

Having thus briefly stated some of the more simple of the general properties of the equations of fluid motion we will now proceed to examine some of the problems which present themselves most naturally to the student. The first case that we shall take up is that of the motion of water in *plane waves* when the excursions of each particle are very small.

When a body of water originally in a state of rest is endowed with a wave motion each particle of the mass has a motion of oscillation or, describes a closed curve in such a manner as to cause the particle of water, after a certain definite lapse of time, to resume its original position on the surface of the wave. By wave, is to be of course, understood simply the *forms* which the water assumes under the action of the disturbing force. Plane waves are those in which the motion of every particle is parallel to a certain fixed plane, and they may be generated by bringing a solid body, *e.g.*, a cylinder, in contact with the surface of the water contained in a rectangular canal of uniform depth, and in such a manner that the line of contact shall be at right angles to the length of the canal.

Plane waves will be generated the instant the contact takes place; these will travel along the entire length of the canal; impinge on the ends and return; we will in our problem, however, limit ourselves at first to the case of a canal of indefinite length and then need not take account of the phenomena at the ends. Our mass of water being then supposed, contained in a canal as described, we will now proceed to the mathematical examination of the waves generated by such a disturbing force as has been mentioned.

Assume the axis of z vertical and positive downwards, the axis of x parallel to the length of the canal, and that of y at right angles to its sides; the origin being on the surface of the water at rest. Let now x_0, y_0, z_0 denote the initial vaiues of the co-ordinates of a particle and let u, v, w denote the displacements which the particle undergoes in the directions of x, y, and z respectively. For plane waves advancing in the direction of x, we have of course $v=o$, and the equation of continuity assumes the form

$$\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dz^2}=o$$

the values of u and w being

$$u=\frac{d\varphi}{dx},\quad w=\frac{d\varphi}{dz}.$$

We have now to express φ as a function of x, z, and t, and, from the nature of the motion, periodic with respect to the last.

The simplest way of expressing this periodicity, will be by introducing in φ a factor which shall be a trigonometric function of the time. We may assume φ in the form

$$\varphi=\frac{\pm\sigma z}{\varepsilon}(x)\begin{Bmatrix}\sin.\\ \cos.\end{Bmatrix}\mathrm{F}(t)$$

when σ is a constant, and the forms of f and F are to be determined, the latter is easily obtained by the following considerations. We know that φ must not change if we increase t by the time of oscillation, denoting this by τ, and φ becomes

$$=\frac{\pm\sigma z}{\varepsilon}f(x)\begin{Bmatrix}\sin.\\ \cos.\end{Bmatrix}\mathrm{F}(t+\tau)$$

Now in order that φ may be a periodic function of t, we must have

$$\mathrm{F}(t+\tau)=\mathrm{F}(t)+2\pi$$

which gives by expansion

$$\tau\mathrm{F}'(t)+\frac{\tau^2}{1.2}\mathrm{F}''(t)+\ldots=2\pi$$

from which since τ is constant,

$$\mathrm{F}''=\mathrm{F}'''=,\ \&c.\ \ldots=o$$

$$\therefore\quad \mathrm{F}(t)=\int_0^t \mathrm{F}'(t)dt\frac{2\pi}{\tau}t$$

and φ now assumes the form

$$\varphi=\frac{\pm\sigma z}{\varepsilon}f(x)\begin{Bmatrix}\sin.\\ \cos.\end{Bmatrix}\frac{2\pi}{\tau}t.$$

Now for the determination of f; substituting this value of φ in the differential equation of continuity $\Delta^2\varphi=o$ and it becomes

$$\frac{d^2f}{dx^2}+\sigma^2 f=o$$

Integrating this and we have for f the equation

$$f=\mathrm{A}\sin.\sigma x+\mathrm{B}\cos.\sigma x$$

when A and B are the constants of integration. This gives us now

$$\varphi=\frac{\pm\sigma z}{\varepsilon}\left(\mathrm{A}\sin.\sigma x+\mathrm{B}\cos.\sigma x\right)\begin{Bmatrix}\sin.\\ \cos.\end{Bmatrix}\frac{2\pi}{\tau}t.$$

This obviously may be written in this form

$$\varphi=\begin{cases}\frac{\sigma z}{\varepsilon}\left(a_1\sin.\sigma x\sin.\frac{2\pi}{\tau}t+b_1\cos.\sigma x\cos.\frac{2\pi}{\tau}t\right)\\ +\frac{\sigma z}{\varepsilon}\left(b_1\cos.\sigma x\sin.\frac{2\pi}{\tau}t+b_2\sin.\sigma x\cos.\frac{2\pi}{\tau}t\right)\\ +\frac{-\sigma z}{\varepsilon}\left(a_1\sin.\sigma x\sin.\frac{2\pi}{\tau}t+\beta_1\cos.\sigma x\cos.\frac{2\pi}{\tau}t\right)\\ +\frac{-\sigma z}{\varepsilon}\left(\beta_1\cos.\sigma x\sin.\frac{2\pi}{\tau}t+a_2\sin.\sigma x\cos.\frac{2\pi}{\tau}t\right)\end{cases}$$

By supposing the constants a, b, ... positive we can, by making the proper ones vanish and establishing certain relations among the remaining ones, obtain an expression for φ which shall contain as a factor the sine of the sum or the cosine of the difference of the quantities $\frac{2\pi}{\tau}t$ and σx. It is of course desirable to introduce the quantity x into the trigonometric factor as the form of φ is alike unaltered if we increase t by the time of

oscillation or x by the wave length. Then making

$$a_1=b_2=a_1=\beta_2=o$$

and for a simple advancing wave making,

$$b_1=a_2=a_1,\ \beta_1=a_2=a_2$$

we have

$$\varphi=\left(\begin{matrix}\sigma z & -\sigma z\\ a_1\varepsilon & +a_2\varepsilon\end{matrix}\right)\sin.\left(\frac{2\pi}{\tau}t+\sigma x\right).$$

We have here before spoken of φ as the velocity function—there is a manifest appropriateness in this case in calling it the *wave function*—a name that we shall adopt for the present.

The quantity

$$\left(\begin{matrix}\sigma z & -\sigma z\\ a_1\varepsilon & +a_2\varepsilon\end{matrix}\right)$$

is called the *amplitude* of the wave, and is evidently the maximum value of φ; τ is the *periodic time*, or *period*, after the lapse of which the values of φ recur; and σx determines the *phase* of the wave at the moment from which t is measured. It is evident that if we have any number of wave functions φ', φ'' . . . which satisfy the differential equations

$$\Delta^2\varphi'=o,\quad \Delta^2\varphi''=o,\ \&c.$$

that this sum must also satisfy the equation

$$\Delta^2\Sigma\varphi=o$$

or any number of wave functions may be compounded into one resultant by simple addition.

Before proceeding to the general problem we will examine the simple case of only one wave function. From the value given above for φ we have

$$u=\sigma\left(\begin{matrix}\sigma z & -\sigma z\\ a_1\varepsilon & +a_2\varepsilon\end{matrix}\right)\cos.\left(\frac{2\pi}{\tau}t+\sigma x\right)$$

$$w=\sigma\left(\begin{matrix}\sigma z & -\sigma z\\ a_1\varepsilon & -a_2\varepsilon\end{matrix}\right)\sin.\left(\frac{2\pi}{\tau}t+\sigma x\right)$$

From these we see at once that the displacements u and w satisfy the equation of an ellipse whose semi axes are

$$\sigma\left(\begin{matrix}\sigma z & -\sigma z\\ a_1\varepsilon & +a_2\varepsilon\end{matrix}\right)$$

$$\sigma\left(\begin{matrix}\sigma z & -\sigma z\\ a_1\varepsilon & -a_2\varepsilon\end{matrix}\right).$$

These values of φ, u and w can however, be further simplified by finding what relation exists between the quantities a_1 and a_2. To find this relation, we proceed to examine the forces which act on any particle of the fluid; these are well known to be of the form

$$-\frac{d^2u}{dt^2},\ o,\ -\frac{d^2w}{dt^2}+g,$$

and the elementary equations of motion thus become

$$\frac{1}{\rho}\frac{dp}{dx}=-\frac{d^2u}{dx^2}$$

$$\frac{1}{\rho}\frac{dp}{dy}=-\frac{d^2w}{dt^2}+g$$

from these we have

$$p=\rho\left\{\int-\frac{d^2u}{dt^2}\,dx+\int-\frac{d^2w}{dt^2}dz+\int g dz\right\}+\text{const.};$$

but

$$-\frac{d^2u}{dt^2}=-\frac{d}{dx}\frac{d^2\varphi}{dt^2}=\left(\frac{2\pi}{\tau}\right)^2\frac{d\varphi}{dx},$$

$$-\frac{d^2w}{dt^2}=-\frac{d}{dz}\frac{d^2\sigma}{dt^2}=\left(\frac{2\pi}{\tau}\right)^2\frac{d\varphi}{dz},$$

therefore,

$$p=\rho\left\{\left(\frac{2\pi}{\tau}\right)^2\varphi+gz\right\}+\text{const.}$$

The constant is evidently p_0 the initial pressure, the first part of the second number of this equation being the increase of pressure at the time t above what it was at the initial instant.

As we suppose ourselves limited to the case of very small motions, we may regard u and w as quantities of the first order; then it is obvious from the form of the expressions obtained for these quantities that σ is of the same order—the other factors being in general of finite magnitude. Then any terms which contain σu, or σw being quantities of the second order, may, with reference to those of the first order of magnitude, be discarded.

MR. E. EASTON, C.E., President of Section G British Association, has issued his address in pamphlet form, accompanied by several of the papers on the same subject—"River Conservation"—which were read at the meeting. The question is an important one, in view of the many disastrous floods which have occurred in recent winters, and information given in this handy form is always attractive.

THE MISSISSIPPI JETTIES.

ABSTRACT OF OFFICIAL REPORT.

[The interest felt by the public, and especially by engineers, in the works undertaken by Mr. Jas. B. Eads, at the South Pass of the Mississippi, prompt us to insert, nearly in full, the report of the Board of Engineer Officers appointed by the President of the United States, under Act of Congress of June 19, 1878, to examine and make a "full report" on the same. The Board was composed of Gen. Barnard, Macomb, Tower, Wright, and Col. Merrill.—EDITOR.]

ARMY BUILDING,
NEW YORK, January, 22, 1879.

HON. GEO. W. MCCRARY,
Secretary of War, etc.:

SIR: The board constituted by Special Orders No. 228, Headquarters of the Army, Adjutant-General's Office, Washington, D. C., October 22, 1878 (of which a copy is annexed, marked 1), assembled at New York on the 11th of December, 1878, and after examination and discussion of the section of the act of Congress by which its proceedings are to be governed, and of other documents referring to their prescribed duties, adjourned to meet at the mouth of the South Pass of the Mississippi on the 30th of December.

The section of the act (section 4, act approved June 19, 1878) requires that—

"The board shall visit the works in process of construction by said James B. Eads, at the South Pass of the Mississippi River, and make an examination of the same, and make a full report of the progress made in the construction of the works, the probable cost of their completion, and the results produced, or that may properly [probably?] be produced by them, their probable permanency, and of the advisability of any modification of the terms of the act under which said Eads is constructing said works, so far as regards dimensions of channel through the jetties, and of the terms of payment for the same; which said report shall be submitted to the Secretary of War, to be presented at the next session of Congress."

The subjects upon which the board is required to make a full report are these:

1. Progress.
2. Probable cost of completion.
3. The results produced.
4. The results that may probably be produced.
5. Their probable permanency.
6. The advisability of any modification of the act, &c.

To discuss these subjects in the order named:

1. PROGRESS MADE IN THE CONSTRUCTION OF THE WORKS.

This requires reference to the original design, which was to construct, starting at marginal points on its banks above where the normal channel depth (30 feet) of the pass itself begins to diminish, dikes or parallel piers (as they are commonly designated) extending thence to the deep water of the Gulf, thus confining the outflowing water in a channel of the same width (nearly) as that of the pass itself until it reaches the deep water of the Gulf and thereby prolonging *through* the "bar" and to the Gulf the normal depth (nearly) which the pass maintains between its natural banks. These parallel piers or dikes, technically called "jetties," are essentially, then, an artificial prolongation of the natural "banks" to deep water in the Gulf. From the land's end (East Point) of eastern shore to 35 feet depth in the Gulf (which was about 300 feet beyond the crest of the bar) was a distance of 11,941 feet, which figures define the length of the east jetty as originally designed and marked out by piles.

The natural bank on the west side of the South Pass extended seaward 4,000 feet farther than the natural eastern bank: the initial point of the west jetty was, therefore, taken about that distance below the origin of the eastern one; hence the required length of the west jetty was 8,000 feet nearly.

The average width between banks of the pass itself is about 700 feet; which width, by the act of March 4, 1875, is fixed as the minimum between the authorized jetties.

The origin of the west jetty was established some distance (about 600 feet) from the west bank; hence it became

necessary to connect this point with the natural west bank by a dam of this length, built at right angles to the jetty, to which the name of the Kipp dam was given.

The original design therefore consisted of "east jetty," 11,941 feet in length (2¼ miles, nearly), a "west jetty" 8,050 feet in length, and the "Kipp dam." The plan of "construction" was, in its main features, essentially that developed by long experience in Holland for dikes, dams, and jetties, on like yielding substrata, viz., a broad foundation stratum of willows or other suitable brush, formed into mattresses" (technically so called), on top of which was built a superstructure of tapering section, of alternate strata of mattresses and stone or gravel.

If we except 330 feet in length of extreme end of the originally designed east jetty, and 280 feet in length of the west jetty (reducing the total lengths of these jetties to 11,600 and 7,770 feet respectively), the jetties, throughout the lengths just mentioned, and the dam have been actually *built up* to the level of average high-water or somewhat above that level, and, owing to subsidence, supplementary elevations of willows and stone have from time to time been added. At present the seaward ends for about 1,500 feet are overflowed at high-tide, and even at low-water through a portion of less extent.

Pages 8 and 9 of the annual report upon the improvement of the South Pass, give for east and west jetty the actual height (July 1, 1878) above "average flood-tide" throughout their whole length. It will be seen that the outer ends and a few points higher up are now at various depths below high-tide. The extreme end of the east jetty is reported by Captain Brown to be 11 feet below; at the date of our visit the lower ends of both jetties for 1,000 feet or more were more or less submerged at flood-tide. Higher up the two jetties were found to be from .3 to 1 foot or 1½ feet above high-tide.

Page 10 gives the same particulars for the "Kipp" dam, which is throughout from ½ to 1-foot above flood-tide.

After the mention just made of "supplementary elevations" added from time to time to the jetties, to compensate for subsidence, it is proper to say a few words on this point.

Owing to the well-known character of the formation at the month of the Mississippi, much subsidence was expected. We have no accurate record of the total settlement of the various parts from the commencement. But on page 19 of the annual report, already cited, Captain Brown has given them for the year ending July 1, 1878. The amount of depression—superficial destruction by storms being eliminated, due to actual settlement *and* compression of willows is, beginning at East Point, one-half foot, gradually increasing to 3⅛ feet at the extreme end of the east jetty.

It is impossible to eliminate the compression of willows to show how much of the above is due to pure *settlement;* but, from the corresponding depression of the heads of the piles along the center portions of the jetties, it would seem probable that it is mainly due to "settlement." On page 2 of the fifth report, Major Comstock gives the settlement of certain piles along the last 1,000 feet of the east jetty, counting from the *old end*, and therefore covering the outer six or seven hundred feet of this jetty as it is at present. Between July 18 and October 21, 1876, three months, the outermost pile had settled 2.55 feet, the innermost 0.67, the gradation being progressive.

On the west jetty the observations began about 270 feet within the present end and extended *back* along 250 running feet of the jetty, the observed settlements graduating from 1.60 to 0.80 feet. Owing, however, to the great subsidence of mattresses and piles at the outer ends, as first fixed, the terminal points were (as before stated) withdrawn 330 and 280 feet respectively, by which withdrawal the jetties now terminate inside the crest of the old bar—the eastern on the very edge, the western at 200 feet inside of it.

Inasmuch as a full statement of the "progress" of the construction can scarcely be made without reference to the repeated partial reconstructions on account of subsidence, we have been led to develop that subject so fully. For details of actual construction, we refer to the third report of Major Comstock, and to map No. 2 of the same report, which exhibits various sections of the jetties as built, and makes it unnecessary that we

should swell this report by further description.

Up to November 1, 1878, there had been consumed in the construction of the two jetties and of the Kipp dam, as stated by the engineers employed by Mr. Eads, 310,830 cubic yards of mattresses and willows and 54,565 cubic yards of stone, mostly small stone.

By decision of the Attorney-General, January 17, 1877 (Ex. Doc. 28, Part 1, H. R., Forty-fourth Congress, second session), the channel through the shoal at the "head of the pass" is made a part of the "South Pass," through which a "navigable depth" is exacted by the act of March 3, 1875. The work at this locality had, however, been undertaken by Mr. Eads simultaneously with that on the jetties. A deflecting dike or "catch-water" was designed and commenced, running from the eastern margin of the entrance to the pass, a distance of about 3,000 feet, in a course starting northerly, curving to the westward; but a channel into the west entrance developing itself, which vessels began to use, the plan was abandoned, and the channel east of the island was closed by a dam. Dikes (called "T-head dams" on charts) were run out from the island and from the west bank, and they now define the present channel or entrance. The island or eastern T-head, about 1,600 feet long (originally 800 feet longer) running northwest by north from the head of the island, consists at present only of a row of piles and a single layer of mattresses on the bottom.

The west T-head, 800 feet distant from the eastern one, starts from a point about 1,200 feet above the origin of the latter and 400 feet distant from the west shore, with which it is connected by a dam. Except the last 350 feet, this T-head is built up above flood tide with five or six tiers of matresses loaded with stone on a double foundation layer of two matresses side by side. The extension of 350 feet has at present its foundation layer only. The dam above mentioned has been built up above high-water level and loaded with stone.

The permanent dam, 550 feet long, extending from the lower part of the island to the east shore and stopping the old east channel, has been built up above high-water and well loaded with stone.

The foregoing described constitute the system of "works" properly belonging to the head of the pass. Auxiliary thereto, mattress "sills," so called, have been laid on the bottom across the two great passes. The one across Southwest Pass runs from the west shore (from which a spur-dam about 400 feet long is first thrown out) to a point near the upper end of the west T-head. The entire length, including spur-dam, is about 3,200 feet. The one across the Northeast Pass runs from the end of the old east dike to the opposite shore, length about 3,000 feet. They consist of a single mattress layer 70 feet wide and about 30 inches thick, weighted with stone.

In these various works at the head of the pass have been consumed 141,100 cubic yards of mattresses and willows, and 10,755 cubic yards of stone, as stated by Mr. Eads' engineer.

The foregoing is deemed a sufficiently full report of the "progress made in the construction of the works."

In what precedes we have made no mention of the wing-dams which have been constructed at various times. Their maintenance not being contemplated as permanent works, it is understood they have already subserved the purpose intended, of accelerating the process of channel formation.

2. PROBABLE COST OF COMPLETION.

Inasmuch as the act of Congress by which the contract was made with James B. Eads for improving the South Pass of the Mississippi River, expressly stipulates that the contractor should have perfect freedom as to the means to be used in obtaining the depths and widths of channel named in the act, estimates of the cost of completing the works could only be made after the board had been officially informed by the contractor of the methods which he purposed using. An official letter requesting this information was sent to the contractor (copy attached and marked A) as soon as the board arrived at Port Eads. His reply (copy attached and marked B), with the accompanying drawings, fully set forth his present plans and methods of completing the works. These plans are necessarily subject to modification, should experience in carrying them into effect

indicate a necessity therefor. Whether such a necessity will arise cannot be foreseen, and therefore estimates must be based on the plans as they now stand.

The following is a brief statement of the work proposed:

1. The top of the east jetty is to be raised to the height which the contractor deems desirable, which varies from 1½ feet above average flood tide at East Point, to 7 feet 9 inches above the same plane at the sea end. The upper part of this jetty, from the point 500 feet below its origin (to which point the jetty may be considered as finished) to a point 9,200 feet below, is to be raised to the level of 1½ feet above average flood tide, and finished by a rounded paving of riprap stone. The next 1,000 feet in length is to be capped by a low wall of rubble masonry. The remaining portion of this jetty to its sea end, a distance of 1,550 feet, is to be capped with large blocks of concrete built in place, on which at a later date a continuous parapet of concrete is to be built. The river and sea slopes of this jetty and its sea end are to be protected by mattresses covered with stone, additionally strengthened at the sea ends and for some distance back by crib-works of palmetto logs filled with stone.

2. The west jetty is to be treated in a similar manner, the changes in the method of finish being made at points opposite those at which the changes are made on the east jetty. The protections designed for the sea end and for the slopes of the west jetty are not so extensive as those for the east jetty, the latter being apparently considered as more exposed to injury.

3. The training-walls at the head of the South Pass (called on the maps "T-head dams") are to be improved. The eastern training-wall, on which but little work had been done, is to be raised above the surface of the water. The portion at the head of the western training-wall, now consisting of piles and one layer of mattresses, is to be completed.

4. The obstruction now in the Southwest Pass is to be increased by the superposition of other mattresses until the cross-section of this pass is made about 12,000 square feet less than it was after the original sill had been laid.

5. The dam closing Grand Bayou is to be maintained by such additional work as may from time to time become necessary.

Mr. Eads estimates the cost of doing the work thus summarily indicated at $349,641.

The board have carefully gone over the details of this estimate, and believe that it is substantially correct. They differ from Mr. Eads in some minor items of cost, but these differences are amply covered by the $58,273 allowed for contingencies. The board is, therefore, of the opinion that the work indicated by Mr. Eads can probably be done for his estimate, provided no extraordinary contingencies intervene.

This at once brings up the question whether the completion of the indicated works is a substantial completion of the original project, and may be so considered in questions of compensation.

When may the works at the South Pass be considered as completed?

A careful study of the act of Congress, under whose authority the original contract was made with Mr. Eads, shows that there is no mention of any specific work to be done by the contractor. The act authorizes him to construct "such walls, jetties, dikes, levees, and other structures, * * * as he may in the prosecution of said work deem necessary."

It also expressly states, that "said Eads shall be untrammeled in the exercise of his judgment and skill in the location, design and construction of said jetties and auxiliary works." The only limitation on the contractor is the provision that the jetties "shall not be less than 700 feet apart."

The whole contract is based on results. Certain specified sums were to be given to the contractor for certain depths of channel obtained by him. It was only required that the jetties should be "permanent and sufficient" "to create and permanently maintain" these various depths, the test of the permanent completion of the work being the creation and maintenance for 20 years of a channel 30 feet deep and 350 feet wide. This is the standard of completion established by law, and there is no power, short of the power that made the law, that can change the standard.

3. THE RESULTS PRODUCED.

The average width of the South Pass

between banks, is about 700 feet; the depth, so long as that width is maintained, is about 30 feet. At East Point, where the eastern bank terminates, and where the width was already increased to 850 feet, the bank confinement ceased; thence to the crest of the bar, 2¼ miles distant, the depth gradually diminished to about 9 feet (average flood tide). The last half mile before reaching the outer crest having, nearly uniformly, only this small depth.

The results produced by the works may be stated as follows (omitting the tabular statement of the report for want of room):

The maximum bar-depth that has been obtained prevailed December 14, 1877, when it was 23.7 feet. At the date of the latest survey, December 28, 1878, it was 23 feet. This slightly diminished bar-depth by no means indicates actual retrogression in the progress of "results." On the contrary, there has been constant progressive general improvement in the jettied channel, at no time more evident than at the present.

At the date last named, a depth of 24 feet, with a channel-width of 300 feet, extended down to within 2,000 feet of the jetty ends; and the same depth with a channel-width of 200 feet, almost to the very ends. Thence to the same depth outside was a distance of but 60 feet with a navigable channel of 23 feet intervening.

The 25-foot channel has nearly the extent of, and not much less width than, the 24-foot channel. From its terminus inside to the same depth outside of the bar, there is but an interval of 160 feet.

The 26-foot channel extends (with a break of only 150 feet) down to within 1,000 feet of the jetty ends. Above the single interruption mentioned, which is 3,000 feet from the ends, the 26-foot channel has in its narrowest parts 100 and 150 feet width; in the widest, 350 and 700 feet, the latter at the site of the so-called "deep hole."

The depth of 27 feet is found at various points in the channel down to very near the jetty ends.

If we compare the above with the chart of a year's earlier date (December 5, 1877) we find a general improvement of navigable channel through the lower 6,000 feet. There is 24 feet where there then was 22 feet; that is to say, a general increment of the channel depth by 2 feet, accompanied by rectification and widening of areas of lesser navigable depths; there being for the 22-foot depth 230 feet width at the bar and a general width within of over 400 feet.

(Even greater increase of depth amounting to 6, 7, and even 10 feet are stated to have taken place in the upper portions of the jetties channel.)

At the *head of the passes* the result of the works has been the procuring of a channel-depth of 22 feet where there was, over the shoal, but 14 or 15 feet. If we compare the present condition with the chart of December, 1877, we find that while the actually navigable depth is not much changed, the distance between the 24-foot curves at T-heads and above the same respectively has increased from 350 to 800 feet; this augmentation of the *bar*-width taking place both inwardly and outwardly, but in much the greater proportion outwardly (*i.e.*, on the Southwest Pass margin). Between the training walls the channel may have somewhat improved, not by increase of maximum depths, but by the diffusion of the current more equally over the intervening space.

4. RESULTS THAT MAY PROBABLY BE PRODUCED.

It is a difficult matter to respond satisfactorily to this requirement of the law inasmuch as the efficient causes cannot be precisely defined or measured. Reference to opinions of engineers who have recommended the resort to jetties and developed their views as to depths which should be obtained by the means proposed, furnishes one basis of judgment. Reference to the *actual results* combined with the progress and present condition of the works furnishes another. The trial of the jetty system at the South Pass, or at least a further study of the subject, before undertaking the construction of a ship-canal, was first recommended in the minority report of the Board of Engineers of 1873, with the expression of opinion that 25 feet at low-water might be attained; the practicability of terminating the jetties inside the bar crest (instead of encountering the great expense and doubtful practicability of

prolongation to deep water) being assumed or supposed probable.

The Board of Engineers constituted by act of June 23, 1874, "to determine the best method of obtaining and maintaining a depth of water sufficient for the purposes of commerce, either by a canal from said river to the waters of the Gulf, or by deepening one or more of the natural outlets of said river," proposed, by the extension of jetties, 900 feet apart, to the depth of 30 feet outside, to obtain provisionally a channel-depth which would, as was estimated, gradually shoal by bar advance in about ten years to 25 feet depth, when the jetties must be extended 1,000 feet seaward to reach 30 feet depth again. The present jetties are about 950 feet apart, and terminate, the eastern almost an the outer edge of the bar, where there was originally but 15 or 16 feet of water; the western, about 200 feet within the outer edge, where there was but 7 or 8 feet of water. The conditions, therefore, by which the engineers of the board of 1874 expected to get, provisionally, 30 feet depth, do not, in the existing arrangements, fully obtain.

Mr. Eads, in his letter to this board, herewith appended, states:

"I believe the natural pass is sufficient to create and maintain a channel through the jetties that shall have a central depth of thirty feet when the jetties are fully consolidated and all leakage through them is prevented; but I do not believe such volume will produce a channel of greater magnitude."

Though the ground for this belief is not here stated by Mr. Eads, it is understood that he relies on the fact that the pass, in its natural condition, maintains a depth of 30 feet in its channel. He believes that the jetties will carry out that depth, undiminished, to the sea. This, too, was admitted or assumed by the board of 1874, *provided always*, the jetties, 900 feet apart or less, be extended to or beyond that natural depth in the Gulf. This proviso is not fulfilled, as we have seen, by the existing jetties.

The foregoing refers to *a priori* opinions. Turning now to the results actually and progressively obtained, coupled with the stage of construction and present condition of the jetties, the facts of the case have been stated under the proper head.

The jetties, since their commencement, have produced an increase of bar (or minimum navigable) depth from 9 to 23 feet; and if the last twelve months have shown no actual increment of that particular element, yet there has been (as already fully set forth) a most decided improvement throughout the whole jettied channel length. There is ground to look for further improvement, coupled with increase of bar-depth, which, to 25 feet, requires the cutting through of a bar of only 160 feet width. But the jetties have not yet (as seen in our statements on "progress, &c.") acquired their full action. The outer ends, though more than once raised, are still submerged. According to Captain Brown, Eighth Report, p. 31, "at least 20 per cent. of the water passing the land's end at East Point, escapes over the jetties and through the meshes of the mattresses at average flood-tide." Mr. Corthell, resident engineer under Mr. Eads, estimates the escape at 25 per cent. By far, the greater portion of this escape takes place along the lower 1,000 or 1,500 feet of length. Moreover, the temporary effect of the operations at the jetties at Grand Bayou, and at the head of the passes, has been to diminish the discharge of the pass by 10 or 12 per cent. The raising and consolidating the jetties at their outer ends will in great measure prevent the loss attributed to imperfect confinement; the volume originally entering the pass at the head may probably be restored.

The foregoing considerations, and the facts already stated under the head of "the results produced," induce us to think that if the jetties were well consolidated and raised sufficiently high to prevent leakage and overflow, a considerable increase of navigable depth would result. We cannot state that in our opinion it is a "probable result" that the depth of 30 feet will be attained, as assumed by Mr. Eads. What the limit will be, cannot be positively announced. That it may attain 25 or 26 feet is all that we can venture to expect, as a depth which shall permanently maintain itself; and, as past experience shows annual fluctuations amounting to about 2 feet, a permanent channel 25 or 26 feet will require an occasional channel of 27 or 28 feet.

With regard to the head of the passes, the considerable widening of the bar or shoal between 24 feet inside and 24 feet outside during the last twelve months, is an unfavorable result. The work designed by Mr. Eads, and embraced in his estimate for "completion," and now commenced, consisting mainly in the raising by additional mattresses the sill of the Southwest Pass, with the view of restoring the lost inflow to the South Pass, and deepening the entrance, may accomplish the result expected. We have not full confidence, however, that that measure alone will do so.

In connection with the "probable results" of jetty construction upon which we are directed to report, there is one to which pre-eminent importance has been attributed and which should not be here overlooked—that of bar advance. The Board of Engineers of 1874, in recommending the jetty construction at the South Pass, assumed that the normal rate (supposed to be 100 feet per annum) would be maintained after the pass was jettied, and hence that, to maintain a depth of at least 25 feet, the jetties must be prolonged every ten years. One of the main arguments used against the resort to the jetty system has always been that a greatly increased rate of bar advance will ensue. On this point we refer to the Seventh and Eighth Reports of Captain Brown, and to his Annual Report of June 30, 1878. By the Seventh Report, page 30, it is shown that over a fan-shaped area of $1\frac{1}{4}$ square miles immediately seaward of the ends of the jetties, there had been between June 20, 1876, and June 22, 1877, a mean "fill" of four-tenths of a foot. Parts of this area had scoured (*i. e.*, had become deeper); in 4 of the 21 sections there had been, on the contrary, a large "fill"; the average total result having been, as just stated, a slight shoaling of 0.4 foot. Over this same area (page 15, Annual Report) there had been in the subsequent twelve months from June, 1877, to July, 1878, a scour (*i. e.*, increase of depth) averaging 1.8 feet.

By the table, p. 33, of the Eighth Report, it is shown that in that portion of the 30-foot curve, lying seaward, between the prolongations of the jetties, an advance of 132 feet had taken place so early as October, 1875, soon after the jetties were commenced and before any impression had been made on the bar. During the subsequent period of two years there had been fluctuations, the advance reaching (July 28, 1877) the magnitude of 242 feet, followed by a retrogression to 108 feet, December 15, 1877. This last was followed by another retrogression (table, page 18, Annual Report) to 60 feet in the early summer of 1878, succeeded by an advance, July 15, to 140 feet.

The foregoing, referring to the portions of the 30-foot curve included between the prolongations of the jetties, affords no proof of *progressive* advance. If, in this connection, we take into account the position of the 30-foot curve outside the jetties (and this is evidently a better test), there is shown, instead of advance, an absolute retrogression. Or, again, if we have reference to deeper curves, Captain Brown's surveys (Annual Report, table, p. 15) show that from June, 1877, to July, 1878, the 40, 50, 60, 80, 90, and 100 foot curves had drawn in towards the ends of the jetties the respective distances of 117, 228, 190, 65, 71, and 183 feet; the 70-foot curve alone showing advance into the Gulf (46 feet). The actual results, therefore, so far as we know them, do not justify the predictions of accelerated bar advance. On the contrary, they show a disappearance of bar material from the front of the jetties.

5. PROBABLE PERMANENCY.

Wave (or storm) *action* of the sea and *decay* or *destruction by the teredo* of the willow mattresses are the principal destructive elements to be mentioned. An additional element of deterioration, viz., settlement not peculiar to the location, but supposed to be so prominent as to involve the question of permanence, must also be noticed.

The jetties, except the extreme ends and contiguous portions, for about 1,500 feet inward, are so well sheltered by shoals that wave action, except on those portions, has little effect. On the sea-ends the effect has been considerable, but mainly superficial, destroying more than once the upper course or courses of mattresses and washing off and scattering the stones (mostly small) which have been repeatedly applied to the top surface. Wave action is by no means as

violent here as in similar exposures on the Atlantic coast. We see no reason to doubt that the thick concrete capping Mr. Eads is now commencing to apply (work having already begun on the upper portions), flanked by enrockments of heavy stones on palmetto-log grillages overlying the original marginal mattresses, will resist sea-action.

Wood of all kinds considerably submerged is sufficiently secure against decay.

Experience here shows that for about 1,700 feet inwards from the jetty ends the teredo destroys rapidly all exposed wood (including in this term the willows of the mattresses) lying more than four or five feet below the surface of the water. Evidence enough of its attacks upon piles and willows exists. But the teredo does not attack wood where the free access of sea-water is impeded. Those portions of a stick buried in mud or sand, or packed around with mud or sand, are secure. We have no reason to believe that the teredo has penetrated or can penetrate far into the interior of the mattress courses; we have pretty good reason to believe that the foundation mattresses are, and will, remain secure; and probably also the bulk of the interior of the masses of willow-work.

In what we have said under the head of "progress" we have given sufficiently full details concerning settlement. It is still very great at the outer ends, though very much less in all those portions more than two or three thousand feet from those ends. That additional superficial applications of stone or concrete will be necessary to the structures we must expect.

In the ordinary sense of the word permanency, *i.e.*, capability of endurance of destructive forces, the works may be said to possess the attribute, to a reasonable degree, for work of the kind thus situated. As regards the outer ends it is yet early to predict to what extent or how long renewals of height to compensate the still progressing settlement must be resorted to.

6. ADVISABILITY OF ANY MODIFICATION OF THE ACT AS REGARDS TERMS OF PAYMENT AND DIMENSIONS OF CHANNEL.

Under this head the board states substantially that, in view of the fact of Mr. Eads having already received $1,686,000 —that Congress has already authorized (by Act of June 19, 1878) advancing $1,000,000 ($686,000 of the total already received being on this account) over and above what, under the terms of the contract has been earned—the board does not admit his plan for further immediate payment; but recommends an additional advance, for carrying on the work, of $250,000.

The board considers it "premature to recommend at this time any changes in channel dimensions, as required by the contract," and "in view of its recommendation that Mr. Eads be provided with sufficient funds to complete his work according to his own programme, and of his expressed ability to obtain the depth and width of channel prescribed" declines to recommend any essential modification. But in compliance with one of Mr. Eads' suggestions it advises as follows:

"As every additional foot in depth of channel is of substantial benefit to commerce, we would suggest the advisability of a change in the terms of payment in the original act, so as to allow of payments for each additional foot gained instead of for every two feet."

SCHWEDLER'S TRUSS.—This truss is so designed as to avoid the necessity for the use of counterbraces. It is symmetrical with respect to the middle of the span. The heights of posts on either side are to be calculated according to the following formula, which is exact and is believed to be new

$$H_x = \frac{Cx(n-x)}{n+\theta x}$$

where n=number of panels

" H_x=height of xth post

" θ=max. ratio of live to dead load

and C is a constant determined for an assumed depth at midspan. If $\theta(n-2)>4$ the value of H_x attains its maximum H before reaching mid span. In this case H is to be used for all posts from this to midspan inclusive. The values of H_x may be laid off above the line of the supports below, part above, and part below or entirely above for roof truss.

WM. M. THORNTON.

A PRACTICAL THEORY OF VOUSSOIR ARCHES. PART II.

By WM. CAIN, C.E., Carolina Military Institute, Charlotte, N. C.

Contributed to VAN NOSTRAND'S MAGAZINE.

III.

GROINED ARCHES.

80. Let ABCD, Fig. 17, be the plan of a groined arch, $\overline{AC}$ and $\overline{BD}$ representing the groins; the elevation is shown, at BMC, of the front face AD. There are abutments at A, B, C and D, one of h is shown at A in plan.

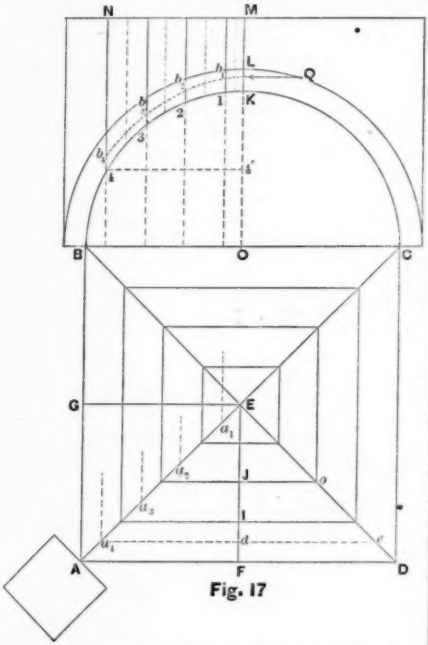

Fig. 17

Let us divide the portion of the arch and load between the groins into simple arches, as AID, a_2 IJ, . . . which rest at their extremities on the groins AE, DE. We can estimate the stability of any one of these arches by principles previously established, and find the resultant pressure that it exercises upon the groin. The latter supports a similar pressure from each side; the resultant of these two pressures, which is generally oblique, can then be decomposed into horizontal and vertical components, which are the forces to be used, in their proper positions, in ascertaining the stability of the simple arch constituting the groin, and also of the abutment against which it leans.

81. An example will render this clearer. The dimensions are given in meters, though any unit may be taken. Let $\overline{AD}=7.54$; the arc AFD in plan, a semicircle whose radius is thus, $\overline{OB}=3.77$; depth of keystone $\overline{KL}=0.47$; the height of surcharge above it, $LM=1.26$. Divide the semi groin AE into a number of equal parts, four in the figure, and suppose, each simple arch, as AID, to terminate at the middle, a_4, of its corresponding division. Project up a_1, a_2, a_3, a_4 to b_1, b_2, b_3, b_4, in elevation. Then on this supposition the weight AIF sustained at a_4 is represented in elevation by MNb_4K, supposing the joints vertical. Similarly for arch a_3 JI, etc. Pass a curve of pressures now through the top of middle third limit at the crown KL and the lower middle third limit about the joint of rupture, taken approximately at b_4 in this case, from which the resultant at $(a_4\ b_4)$ can be found. It is sufficiently correct, and is on the side of safety for the other arcs, as a_3 IJ, to retain the same value and position of Q at the crown. We thus find from the diagram for arc AID the resultants in amount, position and direction at the points $(a_3,\ b_3)$, $(a_2,\ b_2)$, $(a_1,\ b_1)$ of the groins, due to all the arcs in the space AEF.

In the following table of volumes and centers of gravity, v = volume of trapezoid lying just to right of joint to which it refers = width × mean height × $\overline{IF}$.

In this case $IF=JI=0.94$. l is the distance of the center of gravity of the trapezoid from the crown, and m the corresponding moment. V is the volume from the crown to the joint to which it refers found by cumulating the numbers in column v. Similarly M is formed from m, and the quotients $\frac{M}{V}=C$,

give the distances of the centers of gravity of these volumes V from the crown.

Joint.	v	l	m	V	M	C
1	.78	.24	.1872	.78	.1872	.24
2	1.68	.96	1.6128	2 46	1.8000	.73
3	2.04	1.91	3.8964	4.50	5.6964	1.27
4	2.74	2.87	7.8638	7.24	13.5602	1.87
	7.24		13.5602			

Laying off 1m.87 from the crown to the left on Q prolonged and drawing from this point a straight line to b_4, we have the direction of the resultant at b_4. Its amount is found by laying off $P_4=7.24$ on the vertical, through the point 1.87 to left of the crown, downwards, and then drawing a horizontal line to the resultant, which may now, as well as Q, be scaled off. We thus find $Q=5.45$.

Next, combining the forces at a_1, a_2, a_3 and a_4 due to the arcs on either side of the groin AE, we have for the vertical components of the resultants at a_1, a_2, a_3, a_4, 1.56, 4.92, 9., 14.48, respectively, being double the numbers given in column V above.

The horizontal component at each point is $Q\sqrt{2}=7.7$.

82. It is evident that the greater the number of divisions IF, JI, &c., the more accurate the result. It is well to test the above volumes analytically.

Call the radius, OB$=r$, $\overline{KM}=c$, and the variable distance E$d=a_4d=x$. Since any arc as a_4de of the intrados has a radius r, its *rise*, when the half chord is x, $=(r-\sqrt{r^2-x^2})$.

For simplicity we shall introduce one approximation, viz: that the areas of the circular spandrels is nearly that of parabolic spandrels of same encompassing rectangle. This is on the side of safety.

We have, calling $44'=x$,

$$\text{area KMN } 4=\frac{x(r-\sqrt{r^2-x^2})}{3}+cx.$$

On multiplying this by $\triangle x=$ elementary width of a slice, we get the approximate volume of one half of a simple arch, $\| \overline{AD}$, and at a distance x from E. The limit of the sum of such slices, between the limits $x=r$ and $x=o$, gives the total volume of the part AEF

$$\therefore \text{volume AGFE}=\tfrac{1}{4}\text{ groined arch}$$

$$=\lim \sum_{o}^{r}\left(\tfrac{2}{3}xr-\tfrac{2}{3}\left\{r^2-x^2\right\}^{\frac{3}{2}}x+2cx\right)\triangle x$$

$$=\left(\frac{r}{9}+c\right)r^2$$

In this case $r=3.77$ and $c=.47+1.26=1.73$

$\therefore$ vol. AGFE$=30.53$.

From the table we find this volume to be $2(.78+2.46+4.50+7.24)=29.92$, which differs but slightly from the above, and is somewhat less, as it should be.

83. It will conduce to clearness to lay off on an elevation of the groin and abutment, Fig. 18, the forces just found, directly over a_1, a_2, . . . at the same

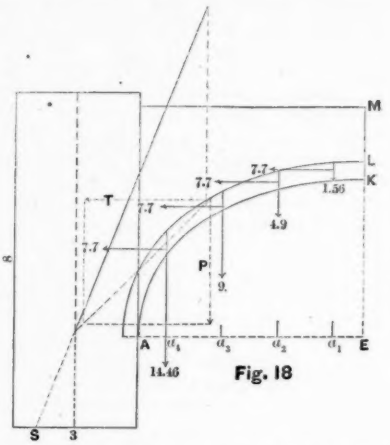

Fig. 18

heights as b_1, b_2, . . . are above the springing, viz., 4.03, 3.76, 3.15 and 2.12. The distance t of the resultant T of the horizontal forces above the springing is thus,

$$t=\frac{7.7(4.03+3.76+3.15+2.12)}{T=30.8}=3.26$$

from which T can be located. Similarly take moments of the vertical forces about A, to find the distance, p that their resultant, P, acts to the right of A.

$$\therefore p=\frac{14.46\times.68+9\times2.01+4.9\times3.35+1.56\times4.69}{P=29.92}$$

$$=1.73$$

The resultant of T and P passes outside of the arch ring above the springing. On combining it in turn with the weight of the abutment $8\times3\times2$, the final resultant cuts the base at s, $\frac{1}{6}$ the depth from the outer edge. The abutment may be increased in size to cause s to pass as near the center as may be desired.

84. The arch ring of the groin in the actual example has a depth of 0.^{m}94, being double that of the ring as drawn; which may thus be supposed to represent its middle half.

To test its stability, combine the resultant of the forces 7.7 and 1.56, being the pressure on the joint midway between a_1 and a_2, with the resultant of the next two concurrent forces, 7.7 and 4.9, to find the resultant on the joint midway between a_2 and a_3; next, combine this last resultant with that of the next two concurrent forces and so on. The final resultant on the springing joint should coincide with the resultant of P and T just found.

The line of pressures is thus found to keep very near the center line down to a_4, below which it passes out of the arch ring, on the extrados side.

The heavy backing will exert horizontal forces to modify this line of pressures, probably keeping it in the arch ring near the springing; for otherwise the intrados joints about the springing must open; but this cannot happen unless the extrados joints open about a_3. If the backing prevents the latter, the former cannot occur; but if no joints open, the line of pressures must lie in the middle third; so that the arch ring is stable.

85. It will be observed, that no pressure at the crown is needed to ensure stability. In fact, if any were supposed it would only cause the final resultant on the springing joint to depart still more from the arch ring.

For other dimensions than those given in this example, a horizontal thrust at the crown of the groin may be needed. For example, when the line of pressures just found falls below the middle third at any joint.

In this case, if we desire that the curve of pressures pass through a particular point in the lower middle third limit, as the one vertically over a_4; let the horizontal thrust, H at the crown KL act at the upper middle third limit. Call the lever arms of H, T and P about the lower point vertically over a_4, h, t and p; we have,

$$Hh+Tt-Pp=o$$

from which H can be found and the curve of pressures located as before.

The true curve keeps within the middle third, and, as before explained, conforms nearly to the maximum and minimum of the thrust in the limits.

86. It is more usual to place the abutments as in Fig. 19. The space between them is usually covered with simple arches as ABCD. The horizontal thrusts

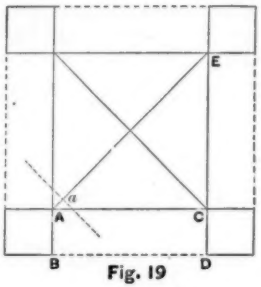

Fig. 19

of the two leaning against one abutment, acting at the crown joints are combined into one, $Q'\sqrt{2}$ acting directly over the center of the abutment, and in the direction of a diagonal, as EA. The weights of the two semi-arches acting at their centers of gravity are combined into one, 2P′, acting at a on the diagonal AE. On drawing now a section of the abutment along AE produced and laying off the forces $Q'\sqrt{2}$, 2P′, T, P, H if any, and the weight of abutment, in their proper positions and combining these forces into one resultant, we ascertain if the center of pressure at the base of the abutment lies within proper limits: the middle third, or whatever limit is chosen from practical considerations. It will be found that the addition of the encompassing arches conduces to stability, the effect of the downward force 2P′, more than counteracting the effect of the force $Q'\sqrt{2}$.

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For definitions of groined and cloistered arches, domes, &c., the reader is referred to Mahan's Civil Engineering; which book like-

give the distances of the centers of gravity of these volumes V from the crown.

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$\therefore$ volume AGFE=½ groined arch

$$=\lim \sum_{o}^{r}\left(\tfrac{2}{3}xr-\tfrac{1}{3}\left\{r^2-x^2\right\}^{\frac{3}{2}}x+2cx\right)\triangle x$$

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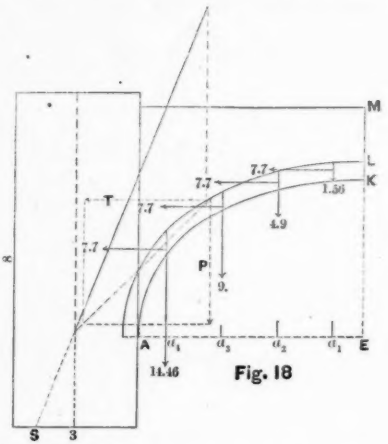

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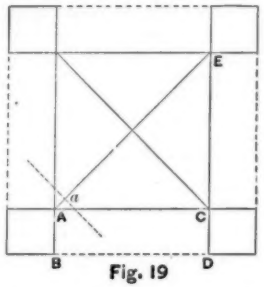

Fig. 19

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For definitions of groined and cloistered arches, domes, &c., the reader is referred to Mahan's Civil Engineering; which book like-

wise contains descriptions and drawings of several noted bridges.

CLOISTERED ARCHES.

88. In the cloistered arch, shown in plan, in Fig. 20, AB, BD, DC and CA, are straight lines, whilst EF is a simple arch

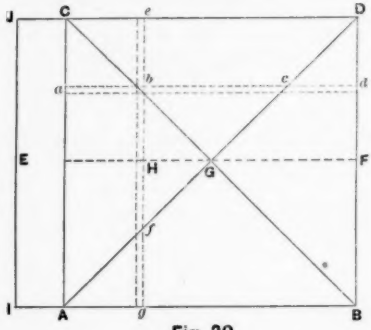

Fig. 20

of span EF, and rise equal to the height of the crown at G above the springing. AD and BC are the groins, forming the reentrant angles on which the smaller arcs, as *ab*, *be*, *cd*, *fg*, etc., meet with an inclined tangent. Thus *ab* is precisely similar in form to the part EH of the full center arch EF. The elements *bc* or *fb* are thus horizontal. Now the thrust at the crown G of the simple arch EF of small width, is horizontal, and is computed as for a simple arch. The arcs *ab* and *cd* sustain at *b* and *c* horizontal thrusts communicated through the horizontal element *bc*. When the centers are struck, the tendency to fall causes pressure on the voussoirs in four directions, ⊥ and ∥ AB, so that the elements, as *bc* and *bf* of the cylinders sustain a uniform horizontal compression in the directions *bc* and *bf*, and the voussoirs composing these elements sustain likewise an inclined thrust (except at the groins, where it is horizontal), in a direction perpendicular to the elements, whose amount is easily determined by the methods affecting simple arches.

89. If the above hypothesis be granted, it becomes an easy question to investigate the stability of a cloistered arch and its abutments, one of which is shown at AIJC.

Thus divide EG into any number of equal parts, and find by usual methods the weights and the positions of their centers of gravity, from the springing AC to any joint, in place of from G to the joint, as hitherto.

Part of the table made out then directly applies to each partial arc, as *ab*. On the elevation of the semi-arch EG and of each partial arch, as *ab*, pass curves of pressures, lying within the middle third, if possible; assuming the horizontal thrust at the top of the arch at the upper middle third limit as a first trial for the arc EF. With the tables made out as above, the resultant at the abutment must be combined, in turn, with the weights from the abutment to the joint considered, to find the centers of pressure on those joints.

We thus find the various horizontal thrusts, acting at the groins CG and AG in a direction ⊥AC. On multiplying each of these thrusts by its vertical distance above the springing, and dividing the result by the sum of the thrusts, we find the vertical distance above the springing at which the resultant of the horizontal thrust T, of the part AGC acts. Similarly, find the horizontal distance to the resultant of the vertical forces, P acting on the part AGC; this resultant representing the weight of AGC. On combining these resultants, T and P acting in the vertical plane EG, with the weight of the abutment, as shown in Fig. 18, we ascertain whether the center of pressure on the base of the abutment falls within proper limits. Since the arc EF causes the greatest thrust, unless the abutment is made to act as one piece, as supposed above, its width should vary, being greatest at E and diminishing to nought, theoretically, at C; the intermediate widths being found in the usual manner from the thrusts of the partial arches resting there. When the backing of the arches can resist a horizontal thrust, the curve of pressures for the smaller arches may be assumed (for safety in designing the abutment) to pass through the centers of the joints at the summit and abutment, or even lower at the summit and higher at the abutment; especially if the stones are not cut to fit snugly.

The writer has not met hitherto with a proper solution of either the groined or cloistered arch, and therefore commends the above to the attention of engineers.

DOMES.

90. The soffit of the dome will be supposed to be generated by revolving a curve about the vertical line representing the rise of the arch called the axis, so that every horizontal section of the soffit is a circle. The extrados may be generated by revolving a similar curve or any other figure about the axis. If we pass two meridian planes, making a small angle, ψ with each other, through the axis, we cut from the dome and backing, if any, a solid FC, Fig. 21, being a part

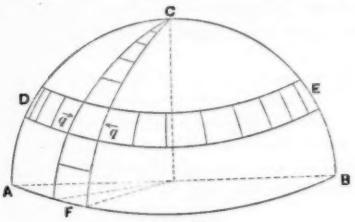

Fig. 21

of a wedge-shaped figure whose soffit is a *lune*. This solid, for want of a better name, we shall call a *lune solid*.

Now pass conical joints, perpendicular to the soffit, at certain distances apart: the part of the dome proper, as DE, lying between any two conical joints, will be called a *crown*.

91. We shall introduce the discussion by a quotation from Dr. Scheffler.*

"The authors who have treated the question of domes (NAVIER: *Résumé des leçons sur l'application de la mécanique*, part 1, No. 349; RONDELET: *Art. de bâtir*, liv. ix, sect. vi, chap. ii, etc,) have commonly divided the dome into lune solids (as defined above), and apply afterwards to these solids the same principles as to simple arches with vertical loads and horizontal thrusts. Now this view is entirely erroneous. It does not make known the influence of the forces which acts upon an arch of this character, and it implies this condition, impossible to realize; that the materials sustain at the summit a finite thrust upon an edge infinitely small.

"It is necessary, on the contrary, to divide the dome ABC, Fig. 21, into crowns, (as defined above), DE resting on the inclined bases of cones of revolution.

"It is proper first to examine the conditions of equilibrium of such a crown; which can moreover form the superior part of a dome open above.

"There are developed in these crowns horizontal pressures q, q, whose directions are normal to the joints of the crown, and more intense in the upper than in the lower crowns.

"When we afterwards consider the lune solid CF, limited by meridian planes, it is necessary to combine the two forces q, q into a single horizontal force Q, acting outwards. It is necessary in all cases that the horizontal thrust at the upper joint may be null.

"This is evident for an open dome; for the dome closed at top, which is only a particular case of the open dome, it results from the fact, that the surface of the joint at the summit reduces to a line, which cannot support a finite pressure."

92. Let Fig. 22 represent a lune solid of the dome considered, and let P_1, P_2,

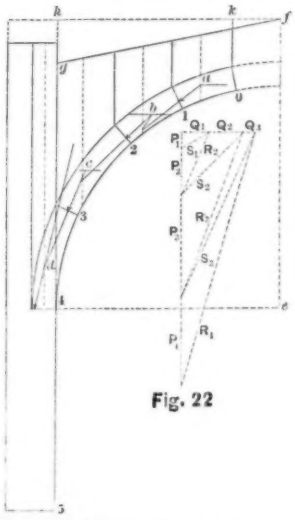

Fig. 22

P_3, P_4, laid off in order on the vertical line P_1 P_5, represent the weights* of voussoirs 1, 2, 3, 4, respectively, with their loads. Let us assume, for the present, that the forces q, q, of the preceding figure act at the centers of the voussoirs; so that the forces Q_1, Q_2 . . , act through the centers of their corre-

* "Theorie der Gewolbe;" also a French translation, "Theorie Des Voutes, &c."

sponding voussoirs, 1, 2, . . . , and horizontally to the left in Fig. 22.

Now the horizontal thrust at joint o is null. The weight P_1 of the first voussoir and load, acts through a and does not meet joint 1, so that there is no stability unless the "crown" including this voussoir, in tending to fall, exerts a horizontal pressure. The resultant Q_1 of the pressures exerted on both sides of voussoir 1 should be so great that when combined with P_1, the resultant shall cut the joint to which it refers, and make with the normal to this joint an angle not less than the angle of friction. These two conditions hold for every joint. If no joints open the resultants will lie in the middle third. Now if Q_1 be made so large that a line drawn through $a \parallel S_1$ the resultant of P_1 and Q_1, satisfies the above conditions, the point where it cuts joint 1 may be regarded as a *possible* center of pressure.

If the above conditions are not satisfied for an assumed value of Q_1, Q_1 must be increased.

Now extend the line through a, just drawn to intersection with the vertical through the center of gravity of the second voussoir and load, whose weight is P_2; from this point draw a parallel to R_2, the resultant of S_1 and P_2, and extend it upwards to intersection b, with the horizontal through the center of the second voussoir along which Q_2 acts. On drawing through b a line to some point on joint 2; a parallel to it, in the force diagram, gives S_2, and cuts off Q_2, as shown in the figure. As before, if S_2 does not make an angle with the normal to joint 2 less than the angle of friction, Q_2 must be increased and the line through b made parallel to S_2 thus found. Similarly we proceed for other joints, until finally we get to a joint, as 3, below which no more forces of the type Q are needed to prevent the resultants on succeeding joints from falling *below* certain limits. The part of the lune solid below joint 3, called the "*joint of rupture*," thus acts as any simple arch; therefore we determine the resultant on joint 4 by combining S_3 and P_4—*i. e.*, by drawing through d a parallel to R_4 of the force diagram, the resultant on joint 4, to intersection with that joint.

Similarly, if we combine the resultant S_3 on joint 3 with the weight of the entire abutment, we find the centre of pressure on joint 5, which should lie within the middle third.

It will be observed that the resultants on joints 1, 2, 3, . . . , are represented in magnitude and direction by the lines S_1, S_2, . . . , of the force diagram, and in position by the arrow heads on the drawing of the arch.

The scale of the arch ring should be as large as can conveniently be drawn, since the lines determining the directions of the resultants are very short, and cannot be well shown on the small diagram. On that account, the preceding direc tions have been made full to conduce to clearness.

93. Scheffler now says, in effect, that if the voussoirs were absolutely incompressible, Q_1, Q_2, . . . should each in turn be the least that will cause stability and should therefore pass through the upper edges of joints 0, 1, 2, . . . ; and the resultants S_1, S_2, . . . should pass through the lower edges of joints 1, 2, etc., if the conditions affecting sliding are fulfilled. (On this point, see art. 106.)

But we know that actual voussoirs are compressible, so that if, as is usual, the actual resultant on the springing joint passes to the left of the center, the outer edge is most compressed, and to allow this the haunches must spread and the top of the arch descend, so that about joint 3 the line of pressures passes below and at the top, above the center line. This is all the more evident if the springing joint opens at the inner edge. In the previous figure, the forces S_1, S_2, . . . , were drawn through the lower middle third limits. Now, if a dome acts like a stone bridge in a lowering of the crown, it would seem that the line of pressures there should lie above the center, so that its most probable position is at the upper middle third limit at the crown, and at the lower middle third limit at the joint of rupture, if the line of pressures can just be inscribed in the inner third of all the joints from the crown to the foundation of the abutment. When the line of pressures can be drawn within narrower limits, it is probably so confined actually.

An illustration of this view is given in article 105 following, which see.

It is plain that the forces q, q do not necessarily act at the centers of the vous-

soirs, as assumed. Their positions are indeterminate. Their least values for the same crown, consistent with no joint opening, corresponds to positions on the upper middle third limit, distant $\frac{1}{3}$ width voussoir from upper joint; their greatest values correspond to the lowest limiting positions.

As it is the object of the investigation to ascertain if the proper thickness of the arch ring and abutment have been chosen, it is well to err on the safe side in our hypotheses. In testing the stability of the abutment, it seems best to consider Q_1, Q_2, . . . , as acting at the centers of the voussoirs; the resultants S_1, S_2, . . . , at the centers of the joints. The latter hypothesis implies a uniform compression on the joints down to the joint of rupture; whereas, in fact, the line of pressures, as explained above, must pass below the center at this joint, giving generally a less total horizontal thrust.

As a modification of the above hypothesis, we may assume that the resultants S_1, S_2, . . . , are tangent to the center line, from the crown to the joint of rupture. It will be found that this involves raising Q_1, Q_2, . . , slightly above the centers of the voussoirs. The construction is much simplified by this assumption which will be illustrated more fully in Art. 100.

94. From the definitions of arts. 90 and 91, and a plan of a voussoir bounded by the two meridian planes whose included angle in arc is ψ, and which is solicited by the two horizontal forces q, q, (acting perpendicular to its vertical faces), whose resultant is Q, we have,

$$\tfrac{1}{2}Q = q \sin \tfrac{1}{2}\psi.$$

If a=half span, and ε=horizontal width of voussoir at the springing, then $a\psi=\varepsilon$. When the angle ψ is small, *i. e.*, when ε is made small enough, we have from the above equation,

$$q = \frac{a}{\varepsilon} Q;$$

from which q_1, q_2, . . can be computed, as soon as Q_1, Q_2, are found by the construction above.

95. *Numerical Example.*—Let us take the half span (Fig. 22) equal to 9.42, the depth of arch ring 0.94; and let the inclined line fg limit the load, the point f being 12.24 above the center e of the soffit, and g, 1.98 lower than f. The radius fk=1.86. Now divide the horizontal hk into six parts, each 1.26 wide, in place of three as before; drop verticals through the points of division, and from their intersection with the extrados draw the joints 0 to 6. We shall suppose approximately that the figures so formed are trapezoids, whose area equals the mean height multiplied by the width.

But now each trapezoidal solid, included between the two meridian planes, has a different thickness. Since the plan of the solid cut from the dome by the two meridian planes is a triangle, if we call its thickness at the middle of the springing joint 1, we find the thickness at the other mean verticals by multiplying 1 by the ratio of the horizontal distance of the mean vertical from ef to the radius of the center of the springing joint about ef. By regarding each trapezoidal solid as having the thickness at its mean vertical, we introduce an error which diminishes indefinitely with ψ, and can thus be made as small as we please.

In the following table, column (1) refers to the joint, column (2) gives the height of the trapezoidal solid, column (3) its width, column (4) its thickness, and column (5) their product representing the forces P_1, P_2, ...

(1)	(2)	(3)	(4)		(5)
1	2.65	1 26	.25	P_1	0.83
2	2.83	1.26	.38	P_2	1.35
3	3.23	1.26	.50	P_3	2.03
4	3.92	1.26	.63	P_4	3.11
5	5.03	1.26	.76	P_5	4.82
6	7.05	1.26	.89	P_6	7.90
7	10.99	0.94	1.00	P_7	10.33

The volumes of the voussoirs and loads can be exactly determined by the principle of Guldinus (see Weisbach's "Mechanics," Coxe, Vol. 1, p. 126), that *the contents of a solid of rotation is equal to the product of the generating surface and the space described by its center of gravity while generating the body.*

For greater accuracy the voussoirs and loads may be considered separately, and their common center of gravity and volume found by combining them afterwards.

The above principle we shall use in some subsequent constructions.

The construction is now proceeded with exactly as described for Fig. 22, which is in fact a drawing to these dimensions.

The induced forces Q_1, Q_2, . . . , were conceived to pass through the centers of the voussoirs. The resultants S, S_2, . . . , on the joints were made first to pass through the lower middle third limits, and afterwards through the center of those joints. In both cases the joint marked 3 in Fig. 22 was the joint of rupture: the resultant on the springing joint, in the first instance, coinciding with the resultant as drawn in Fig. 22; in the last case passing nearly through the extrados. The total horizontal thrust in the first case=7.75; in the last, 8.13. If we take the width of abutment at 3, its height above the springing 10.99, its depth below it 7.01; its mean thickness is $\frac{10.92}{9.9}=1.1$, and its total volume, including a part of the arch is $3\times18\times1.1=59.4$; which combined with the resultant S_3 on the joint marked 3 in Fig. 22 cuts the base, for the first case noted above, only 0.05 outside of the middle third, in the last case 0.18 outside.

The force $Q_2=2.3$ is the largest of the forces Q_1, Q_2, . . . , whence by art. 94, $q=\frac{a}{e}Q_2=9.9\times2.3=22.77$ cubic units of stone. The force q acts on an area of 1.22 square units. There is evidently no danger of crushing from the horizontal thrust around the second crown from the top, as stone will bear on a square unit a pillar of a square unit section and several thousand units high. Similarly for the resultants on all the joints. We conclude that with the backing used, or by increasing the depth of arch ring at the springing about one-third, the arch will be stable. A very slight increase in the width of abutment will prevent any joint from opening.

In the preceding example any unit of length may be taken, as foot, meter, etc.

96. We took the thickness of voussoir at springing as unity. Since ψ should be a very small angle for greater accuracy in the assumptions about center of gravity and the values of q, etc., it may be thought that the thickness is too great. But since multiplying the weights of voussoirs and abutments by the same quantity does not change their ratios, the thickness taken is really immaterial except in finding q, since the centers of gravity are assumed to lie in the same verticals on the vertical projection of a medial meridian section, no matter what the thickness may be.

In truth, the assumption about the position of the center of gravity of a voussoir and load is more nearly realized when the thickness is appreciable.

97. When the soffit and exterior surface of the dome are both surfaces of spheres having the same center A (Fig. 23), the volumes of the voussoirs are

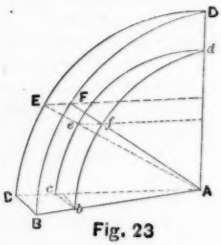

Fig. 23

easily obtained. Thus let r=AD=radius of outer sphere; one half its volume is $\frac{2}{3}\pi r^3$. The wedge ABCD is $\frac{1}{n}$ of this volume, if BC is $\frac{1}{n}$ of the circumference $2\pi r$ $\therefore$ volume of wedge ABCD=$\frac{2\pi r^3}{3n}$.

Now divide the altitude AD into m equal parts (2 in the Fig.) and pass horizontal planes through the points of division; the surface of the lune BCD is divided into m equal parts, by geometry, and the pyramids formed on these parts, as bases, with the center of the sphere as the common vertex, are therefore equal.

The volume of such a pyramid, as A—BCEF, is consequently, $\frac{2\pi r^3}{3mn}$.

Similarly the volume of a pyramid of the sphere, whose radius, $r'=\overline{Ad}$ is that of the soffit, and which is bounded by the same planes as any one of the preceding pyramids is, $\frac{2\pi r'^3}{3mn}$, since these pyramids, as A*bcef*, are diminished images of the preceding ones; every corresponding line being diminished in the ratio of r' to r. It follows that if

horizontals are drawn through the ends $f, \ldots,$ of the edges of the second set of pyramids, they will divide the distance $\overline{Ad}$ into m equal parts. The same holds for similar pyramids of any sphere; so that if the center line of the arch ring on the sectional elevation, as de in Fig. 24,

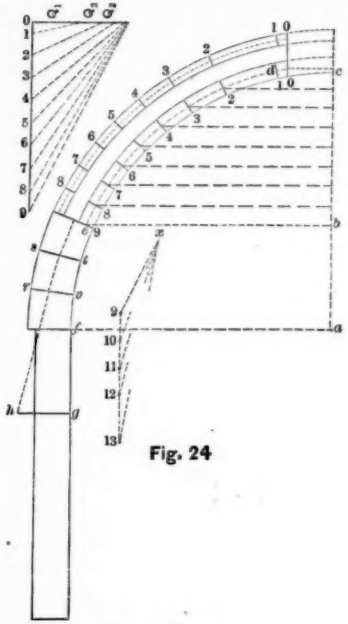

Fig. 24

is divided into equal parts and horizontals be drawn through the points of division to de and the joints be drawn through the latter points and the center of the sphere, the voussoirs so formed are all equal in volume.

Recurring to Fig. 23, we see that the volume of the voussoir Be = difference in volume of the two pyramids $= \frac{2\pi}{3mn}(r^3 - r'^3)$.

98. In the open dome however, as Fig. 24, it is not always convenient to divide the rise ac into such equal parts that certain of the points of division will lie on the horizontals through d and e. We proceed then as follows: By geometry the area of the zone formed by revolving an arc as rs (Fig. 24) about the rise ac is equal to the altitude h of rs multiplied by $2\pi r$, r being the radius $\overline{as}$ of the sphere. Pass now two meridian planes through ac, whose included angle is $\frac{1}{n}$ of a circumference. The part of the zone included between them has an area $\frac{2\pi rh}{n}$; so that the pyramid formed on this base with a vertex at a, has a volume $\frac{2\pi r^2 h}{3n}$.

Similarly the pyramid having the part of the zone represented by tv as a base, has a volume, $\frac{2\pi r'^2 h'}{3n}$ where r' = radius at and h' = altitude of arc tv. Therefore the volume of the voussoir $rstv$ included between the meridian planes and the conical joints rv and st is,

$$V = \frac{2\pi}{3n}(r^3 h - r'^3 h')$$

where r and h are the radius and altitude of the exterior arc, r' and h' of the interior. As before shown, if the altitudes of the type h are made equal in successive arcs, the values h' will all be equal. The divisions into equal altitudes can best be made along the center line in consequence of what follows.

99. Let us refer again to the inner dome de (Fig. 24).

If we pass horizontal planes midway between the horizontals drawn, also pass conical joints through their intersection with the center line de, we divide the previous voussoirs exactly in half, so that *the centers of gravity* of the first voussoirs lie on these supposed intermediate conical joints. They also lie nearly on the center line de, and the error of so regarding them can be made as small as we choose by sufficiently diminishing ϕ, the angle included between the two meridian planes and the height of voussoir.

The centers of gravity of the voussoirs will therefore be assumed to be on the center line of the elevation of the medial meridian section de, at the intersections of the horizontals drawn midway between the first horizontals drawn that divide bc into equal parts.

100. Fig. 24 represents a meridian section of the Church of St. Peter, at Rome. The dimensions given by Scheffler, as I understand them, are as follows:

The radius of the soffit is 72 feet, and of the outer surface, $\overline{as} = 83.8$. At 31

feet above the springing, the structure is composed of two domes, the outer having a thickness of 2.6 feet, the inner being 4.1 thick at d and 5.1 at e, so that the center line de is described from a center slightly below a on the vertical $\overline{ac}$ produced. The dome has an opening at top 12.4 radius, and the lantern supported at the top is equivalent in weight to a block of stone 2.1×56.6, of which the outer shell supports one-third and the inner two-thirds. The first voussoir at the top, in both shells, is made 2.1, horizontal width; the altitude of the center line, $\overline{bc}$, for the part de, is then divided into 8 equal parts and the joints drawn as in the figure, a similar construction applying to the outer shell. The part below the two shells is similarly divided into 3 equal parts. Applying the formula just deduced in Article 98, measuring the altitudes on a drawing to a scale of 4 feet to the inch, we find the volumes of the voussoirs like $rstv$, $\frac{2\pi}{3n}$ 28696, the voussoirs of the outer shell, $\frac{2\pi}{3n}$ 3957, except the top one, whose volume is $\frac{2\pi}{3n}$ 359. cubic feet. The volume of the top voussoir of the inner shell is $\frac{2\pi}{3n}$ 501 cubic feet. The voussoirs 2 to 9 of the inner shell, were each, in turn, assumed to have an outer surface concentric with the soffit, of radii equal to the mean radii of the outer surface for the voussoir considered, *i.e.*, equal to 72 + mean thickness in feet of voussoir. We thus find the volumes of voussoirs 2 to 9 equal to the constant $\frac{2\pi}{3n}$ multiplied in turn by 4695, 4805, 4915, 5000, 5124, 5206, 5267 and 5400.

The volume of the lantern, by the law of Guldinus (art. 95) $= 2.1 \times 56.6 \times \frac{2\pi \times 13.45}{n} = \frac{2\pi}{3n}$ 4794, one-third of which is added to the volume of voussoir 1 of outer shell and two thirds to that of voussoir 1 of the inner shell. The part fgh, 10.3 wide and 23.7 high, has a volume, $10.3 \times 23.7 \times \frac{2\pi \times 77.15}{n} = \frac{2\pi}{3n}$ 56500. This part has not the full width of the bottom voussoirs as drawn in the figure.

We now lay off on vertical lines, the weights just found, omitting the common constant $\frac{2\pi}{3n}$.

The loads affecting the outer shell are laid off to its left; those pertaining to the inner shell just below its center about (not shown in figure).

To be on the safe side, we shall assume that the resultants on the joints from the summit to the joint of rupture are tangent to the center line of the ring. Thus for the outer shell, draw through the points 1, 2, inches, of the force lines, parallel to tangents to the center line at joints 1, 2, . . . (or $\perp$ to radii). These lines cut off successive distances on the horizontal through o, equal to the radial forces Q_1, Q_2, . . . exerted by the successive crowns 1, 2, . . .

We find that below joint 6 there is no longer a radial force needed; so that below that joint the curve of pressures is continued to joint 9 as in a simple arch.

Similar results were found for the inner shell. The centers of pressure on joints 9 of the outer and inner shells are at the outer middle third limit for the outer shell, and slightly above the center line for the inner shell. This necessitates spreading about joints 6, so that the line of pressures there is below the center line, so that the actual horizontal thrust is less than estimated, as stated above.

Now combining the resultants at joint 9 into one, laying off $\overline{x9}$ equal and parallel to it, its position being found by moments, we continue the line of pressures as per dotted line to joint hg. The successive volumes of voussoirs are laid off on the force line 9 . . . 13. This second force diagram is drawn to a smaller scale than the preceding.

101. At joint hg this line of pressures passes outside the joint so that the dome cannot be regarded as sufficiently stable in itself. If rotation occurs the line of pressures would approach the extrados at the summit, the intrados at the joints of rupture and the extrados at joint hg.

By encircling the dome just above the springing by a band of iron of sufficient cross section, stability may be assured. The band may be applied above the springing if desired. It evidently is much less effective in preventing de-

formation of the dome if applied below the springing as was done in this case.

The total horizontal thrust of the lune solid (being $\frac{2\pi}{3n}$ × the horizontal component of $\overline{x9}$) is $Q=39600\,\frac{2\pi}{3n}=13200\phi$ cubic feet of stone $=924.\phi$ tons, if we put the weight of a cubic foot of stone at 157 lbs. $=.07$ ton.

If this is to be entirely destroyed by the iron band, so that the resultants below the springing will all be vertical, we have the strain on the band by art. 94, when ϕ is small

$$q=\frac{Q}{\phi}=924 \text{ tons.}$$

Now iron, exposed to a dead strain alone, may safely be subjected to a strain of $7\frac{1}{2}$ tons per square inch; so that the bar may have a cross section of 123 square inches.

If the iron stretches $\frac{1}{12000}$ of its length for every ton per square inch, the ring whose diameter is 168 feet will elongate 0.33 feet, so that the diameter is increased 0.04 feet. There will consequently be a slight deformation of the arch, in consequence the top of the abutment moves slightly outwards, and the pressure on its base is not generally vertical; *i. e.*, the iron band has not totally destroyed the horizontal thrust. The action of the band is like that of a radial force acting inwards and equal, or nearly equal, to the total horizontal thrust.

If the hoop encircles the dome just below the springing of the two shells, it will prevent deformation of the arch also; for there can be no spreading outwards at this point, any tendency that way being met by the resistance of the hoop, which thus supplies sufficient horizontal force to force the line of pressures below it to keep within the joint areas a certain distance, dependent upon the spreading of the arch at the hoop. If this spreading is inappreciable, then the hoop exerts force enough to restrain the line of pressures to the centers of the joints nearly below it.

There is therefore no necessity in the voussoir dome for additional hoops below the first, unless the first is unable to destroy the outward thrust. The problem is then really indeterminate of ascertaining the precise amounts of the strains sustained by the several hoops. The one nearest the joint of rupture of course will sustain by far the greatest part; the hoops, at joints where no spreading would occur, if they were not applied, not sustaining any. The resultants on the abutment joints cannot approach the outer limits without the top of abutment moving outwards; as this is prevented by the top hoop principally, it is evident that the one hoop should be placed not far below the joint of rupture. It would seem best to make these hoops of steel as it does not stretch as much as iron. Wire cables with a means of tightening would be especially convenient.

102. We see that the thrust of the type Q is greatest on voussoirs 1 of both shells. Thus for outer shell, Q_1

$$=16900\,\frac{2\pi}{3n}=5633\ \phi$$

Thus for inner shell, $Q_1=6000\ \phi$, which multiplied by .07 gives the thrusts in tons. By art. 94, we have $q=\frac{Q_1}{\phi}$ or 394 tons and 420 tons respectively. Now voussoir 1 of outer shell has an area of 6 square feet; the lower voussoir 1 an area of 10.45 square feet, so that the pressures per square foot are 66 and 40 tons respectively, which good stone can stand easily.

The thrusts Q_1 of both shells is probably less than assumed, for the force diagrams indicate a small value for Q_2—in fact for the lower shell Q_2 nearly vanishes—but the compression around the ring of the first crown would necessarily bring the second more in action thereby increasing its thrust. The tendency then is to equalize more nearly the values of the thrusts Q_1, Q_2, . . . than as given by our construction.

103. It is evident that the greatest economy is subserved, by the employment of one or more thin domes to a short distance below the joint of rupture, as is the practice generally in large domes. One shell would suffice if the weight of lantern (if any) could be carried by it; otherwise two or more should be used.

104. The abutment below joint *hg* is counterforted so as to present a greater width than shown in the figure. The introduction of the hoop of course prevents any movement in it, so that the

stability of the whole structure is assured.

105. We shall give now the spherical dome closed at top to illustrate the view taken in art. 93 of the position of the actual line of pressures, besides other points not noticed before. This dome, Fig. 25, has a thickness of one fifteenth the span.

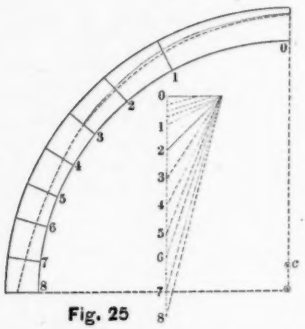

Fig. 25

Divide the altitude of the center line of the ring into eight equal parts, draw horizontals &c. as before. The lune solid is thus divided into eight equal parts which lay off on the force line 0 . . . 8.

Now a dome of this kind fails by rotating about the outer edges of joints at the crown and abutment, and the inner edges about joint 4; each lune solid separating from the others and acting as simple arches.

For a dome then of small stability the line of pressures passes nearly through the top, the intrados edge at joint 4 and the outer edge at joint 8.

For a dome of greater stability its position depends upon the amount of spreading at the haunches, and the consequent rocking at joint 8. If a line of pressures can just be inscribed in certain limits, equally distant from the center, then it is probable, from a consideration of the way in which an arch settles, that the actual line of pressures touches these limiting curves towards the extrados side at the top and abutment, and next the intrados side at the joint of rupture.

Now if such a curve can be inscribed in the middle third no joints will open, and we may conclude that the dome has sufficient stability.

Draw an arc of a circle through the upper limit at the summit and the lower limit at joint 4 with a center c and assume that this arc coincides with the line of pressures a certain distance from the top, the resultants being tangent to it.

Then at some joint as 2 continue the pressure line down to the springing with the horizontal thrust found at 2. If the line so found does not keep within the middle third, let it be commenced at another joint, until one is found that will satisfy the conditions. On dividing voussoir 2 into four others, it was found that a line of pressures, continued as for a simple arch, from where the arc above cuts the upper joint of voussoir $1\frac{3}{4}$, keeps almost entirely in the middle third, as shown by the dotted line: cutting joints 4 and 5 at the lower limits and joint 8 at the outer limit.

This is therefore a *probable* curve of pressures as determined from considerations of how an arch settles.

106. The construction given in art. 93, gives the lines of pressures, that beginning at the summit, first makes Q_1 a minimum, and then, for the same line of pressures in order, Q_2, Q_3, . . . *But this does not make, necessarily, the total thrust,* $Q_1+Q_2+\ldots$, *in the lower part of the arch a minimum.* In fact, this total thrust determined in this way, restricting the line to the inner third is found to be $\frac{1}{4}$ to $\frac{1}{3}$ greater than the thrust, determined as follows, that corresponds to the *minimum of the total horizontal thrust,* $Q_1+Q_2+\ldots$, within the limits taken; as first given, in effect, by Prof. Eddy, in his "New Constructions in Graphical Statics." Take the upper middle third limit as the line of pressures down to a joint ($1\frac{1}{8}$ in this case), where the horizontal thrust may become constant, as for a simple arch, the pressure line below this point remaining within the inner third and just touching the inner limit at some point. The line so drawn coincides nearly with the first below joint 3. As mentioned in art. 93, the thrusts Q_1, Q_2, . . . will, by this construction, be really slightly outside of the inner third limits. The point where the simple arch begins is higher than in the previous cases. If the center line of the arch ring is assumed for the line of pressures, a certain distance from the summit the joint of rupture is lowered,

i.e., a less part of the lune solid acts as a simple arch, and the horizontal thrust is increased.

If the curve of pressures is taken to coincide with an arc starting at the upper limit at the crown as before, and lying below the previous arc, the thrusts Q_1, Q_2, . . . near the crown are lessened; but the total horizontal thrust will be greater than as found in art. 105, as is evident.

Now, of all such arcs it is impossible to say which one, if any, is the true line of pressures, since this is a function of the deformation of the arch.

It may be remarked further that the direction of the thrusts near the summit are most likely more inclined than drawn above, since, by the construction above, a very small crown of voussoirs next the summit exerts a comparatively large thrust; so that the upper crown is compressed sufficiently to bring the next crown more in action, and so on down. If we divide voussoir 1 into four equal ones, we find that the circumferential thrusts *per square unit* on each crown going from the top are proportional to 2.5, 2.3, 1.9, and 1.8 respectively, so that the unit strains decrease going from the summit as stated. It would certainly give a large stability to apply the method of art. 100 and require that the line of pressures so drawn should keep within the inner third below the joint of rupture. The arch cannot fall unless the line of pressures nearly touches the contour curves; hence, of the infinite number of positions it can take, it would seem that the first just drawn should offer sufficient stability, though it may not be exactly the true one.

107. If there is a weight at the summit, since its effect is to move the point o of the force line upwards and thus increase the thrusts in the top crowns, the line of pressures with a constant horizontal thrust must commence nearer the summit than before. The reverse happens when any weight is taken from the top of the dome, as for an open dome. In the former case, a small weight at top will cause nearly the whole of the lune solid to act as a simple arch.

CONICAL DOME.

108. Let Fig. 26 represent a meridian section of a conical dome. Divide the

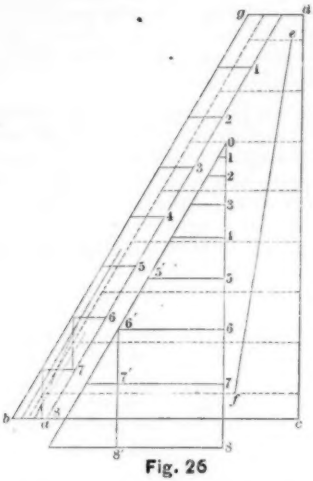

Fig. 26

altitude $\overline{cd}$ into eight equal parts, and pass horizontal planes through the points of division giving the joints 1, 2, . . . Midway between these joints draw the horizontal radii of the center line shown by the dotted lines. If we call the horizontal thickness of the ring $ab=t$, the vertical distance between any two joints h, and the mean radius of the center line between these joints by r, we have for the volume of the voussoir included between the two joints and two meridian planes, making an angle ψ with each other,

$$V=htr\psi,$$

according to the law of Guldinus.

The weights of the successive voussoirs vary therefore as r. Therefore lay off on the vertical force line, 0 . . . 8 successive distances, proportional to the dotted radii, beginning with the first voussoir. The line $\overline{ef}$ cuts off $\frac{1}{4}$ of these radii counting from $\overline{cd}$.

Next draw the line $\overline{06'} \parallel \overline{bg}$ and pass horizontals through the points 1, 2, . . . of the force line. If the resultants on the joints are assumed to coincide with, or be parallel to the center line, the hypothenuses of the triangles just formed represent the strains on the joints. Thus $\overline{06'}$ is proportional to the strain on joint 6, and $\overline{66'}-\overline{55'}$=horizontal radial force Q exerted by the sixth ring.

Now if the arch is assumed practically undeformable, the center line is the line of pressures, and the force diagram is sufficiently correct.

For compressible materials however, if the top and bottom are kept from moving horizontally, the middle of the dome from the compression of its rings tends to move inwards, which requires that the line of pressures there lies outside of the center line, and, as a consequence, inside the center line at top and bottom. Its exact position is indeterminate and is dependent upon the method used in building (whether with or without centers or supports) also on the fitting of the stones. If the abutments spread, as probably happens, the wedge-shaped solids, included between the meridian planes, tend to separate next the base; and the inner edges of joints next the abutment must bear the most, *i.e.* the line of pressures there is inside of the center line.

We see, therefore, that the assumption that the center line is the line of pressures, is on the side of safety, so that the force diagram above may be used in determining the dimensions of abutments or hoops to withstand the horizontal thrust.

109. The least horizontal thrust, consistent with no joints opening, may be found as follows (also see Eddy's New Constructions, &c.):

Assume the centers of gravity of the voussoirs to lie on the center line midway between the joints, which hypothesis can be made as near the truth as we choose by sufficiently diminishing ϕ. Now combine the thrust at joint 6 , 06′ (as found above), supposed to act at the exterior middle third limit, with the weight of voussoirs below it; if the resultant, $\overline{08'}$, strikes the base at the inner limit, the horizontal thrust, $66' = 88'$, is a minimum in order that no joints open. The dome, however, may be perfectly stable when joints open, so that the smallest thrust consistent with stability is less than the above. In the above figure it was found, on a second trial, that at joint $6\frac{1}{4}$ the horizontal thrust first became constant, so that the part of the dome below it exerted no circumferential thrust on the hypothesis. The point of contact with the outer limit is probably nearer the middle of the dome section, as the yielding is greatest there.

The two constructions given thus indicate limits between which the true thrust is found.

110. The preceding construction is the concluding one for this series.

In what has preceded we have been careful to state clearly the hypotheses which are introduced, and to criticise them in the light of both theory and facts.

It is not difficult for a mathematician, having made certain assumptions, to develop, perhaps, an elegant theory, quite dazzling to the inexperienced; but the engineer requires that the hypotheses be proved correct, or at least nearly so, before he can use them in practice.

So far from enforcing an unproved hypothesis by confident assertions, the writer may have leaned too much the other way by too often employing the word "probable" where a stronger adjective would have better suited, especially in the parts referring to true curves of pressure. It is evident that the difficulty of locating the real pressure curve in a simple arch is increased for its combinations in a much greater ratio.

It has been pointed out, however, that the arch may satisfactorily be tested without its true position being known, from considerations of arches at the limit of stability. In fact, it is believed that this true position is closely approximated to by drawing a curve of press ures, within limits, approximately equidistant from the center line, corresponding to the maximum and minimum of the thrust (see art. 27).

This principle receives corroboration from certain constructions pertaining to the solid arch, which apply to the voussoir arch when the stones are cut perfectly, the mortar joints very thin, and no joints open, the spandrels not being supposed to exert any resistance.

In such cases, the pressure curve for the solid arch is identical with that for the voussoir arch.

This view will be clearly exposed in a paper that will follow this, on "Solid Arches," to which the reader is referred.

THE AMERICAN IRON TRADE IN 1878.

From "The Bulletin."

We are again placed under obligations to our special correspondents for information which enables us to give our usual annual summary of the condition of our iron and steel industries at the beginning of a new year. The information thus obtained has been analyzed with care, and so much of it as relates to pig iron has been embraced in a table, which is herewith presented, in connection with comparative figures for 1877, which are taken from our last annual report. As more detailed information than has yet reached us will be received in due time from individual producers, it is to be expected that the figures of production, stocks, etc., which will appear in our next annual report will be slightly different from those of the table; but the general results in districts and States, and in the country at large, will not be materially altered. The figures now given in the table and in the remainder of this summary are partly estimates, but they are estimates made not by us but by practical ironmasters, each one of whom testifies only concerning his own district and of his own knowledge. We are thus particular in giving all the facts which affect the tabulated and other statements which are here submitted, that the trade may know what degree of credibility to attach to these statements. We think we are justified in claiming for them substantial accuracy in all essential particulars.

Pig Iron.—The production of pig iron in the United States in 1878 was about 70,000 net tons (2,000 pounds) in excess of the production of 1877. In 1877 there were produced 2,314,585 tons, and in 1878 the production was 2,382,000 tons. A reference to our table will show which States increased and which decreased their production during the past year. Pennsylvania shows an increase of over 100,000 tons, while Ohio shows a decrease of over 30,000 tons. In 1878 Pennsylvania made more than 50 per cent. of the total production of pig iron in the United States.

The pig iron produced in 1878 we classify as follows according to the kind of fuel used, and give comparative figures for preceding years:

Kind of Fuel.	1878.	1877.	1876.	1875.
Anthracite	1,039,000	934,797	794,578	908,046
Bituminous.....	1,093,000	1,061,945	990,009	947,545
Charcoal.....	250,000	317,843	308,649	410,990
Total	2,382,000	2,314,585	2,093,236	2,266,581

At the close of 1877 there were in the United States 716 blast furnaces, of which 270 were in blast and 446 were out of blast. At the close of 1878 there were 700 furnaces, of which 260 were in blast and 440 were out of blast. These figures, taken in connection with those of production above given, indicate an increased average production of the active furnaces in 1878 over 1877. During 1878 there were 18 furnaces torn down, burned down, or otherwise taken out of the active list, and there were two new furnaces erected—one in Ohio and one in Tennessee, showing a net decrease in the year of 16 furnaces.

The stocks of pig iron on hand and *unsold* at the close of 1877 amounted to 642,351 net tons. At the close of 1878 they were very much less, about 516,000 tons. At the close of 1876 stocks amounted to 686,798 tons. These figures show a decrease in stocks of 44,447 tons from 1876 to 1877, and of 126,351 tons from 1877 to 1878. In the whole of Pennsylvania there was a decrease in 1878 of about 30,000 tons, although in the Lehigh Valley there was an increase of about 13,000 tons. There was a decrease in New York of about 24,000 tons; in Ohio of about 17,000 tons; and a marked decrease in Michigan, Missouri

and some other States. The shrinkage in stocks was remarkably uniform in all iron-producing States, and it is very significant of the caution which characterized this branch of the iron trade throughout the year. No State materially increased its stocks in 1878.

The pig iron on hand and unsold at the close of the year, and at the close of other recent years, was made with fuel as follows:

Kind of Fuel.	1878.	1877.	1876.	1875.
Anthracite.....	192,000	239,493	268,122	274,743
Bituminous..	150,000	156,818	174,302	165,482
Charcoal.....	174,000	246,040	244,374	320,683
Total.....	516,000	642,351	686,798	760,908

tion to a correct result is not difficult. In 1878 production was increased about 70,000 tons and stocks were decreased about 126,000 tons. As our imports of pig iron in 1878 did not vary greatly from the imports of 1877, and as our exports of pig iron were less, and as we have failed to detect any speculative movements in 1878 that would withdraw large blocks of pig iron from the market, we think it entirely safe to assume that we increased our consumption of pig iron in 1878 over 1877 about 195,000 tons.

Manufactured Iron and Steel.—It follows from what has just been said that the rolling mills of the country were more steadily employed in 1878 than in 1877. But of this there is abundant and positive evidence additional to that which is based upon the increased production of pig iron and the decrease in pig iron stocks. Pittsburgh rolling mills and steel works were more generally employed in 1878 than in 1877. Iron shipbuilding is known to have been more active in 1878 than in 1877. The elevated railroads of New York have required a large quantity of finished iron. Iron bridge building, on home and foreign account, has shown an improvement over 1877. Government and other public buildings have been pushed toward completion, and have required many thousand tons of iron. There was a large increase during the year in the number of locomotives and railroad cars manufactured. There were more miles of railroad constructed in 1878 than in 1877. Ten out of eleven of our Bessemer steel works were busy during the whole year in the production of steel rails, and the manufacture of iron rails during the year was certainly as active as in 1877. Other steel works are known to have been busy. The consumption of old iron rails in 1878 is known to have been quite large, both in iron rail mills and in other rolling mills. The prosperity of our farmers, who have had two good consecutive crops, for which they have found a market, and the general revival of business throughout the country in the latter half of 1878, were influences which favorably affected the iron trade of the country. The railroads were generally well employed throughout the year.

The consumption of pig iron in 1878 was apparently greater than in 1877. Production was greater and the reduction of stocks was also greater. There are unknown quantities that elude the grasp of the statistician in attempting an estimate of the consumption of pig iron in any given year, so that an exact statement is never possible; but with the two most important elements of the problem given —production and stocks—an approxima-

Iron and Steel Rails.—We have above referred to the improvement in this branch of the iron trade during 1878, but we now add more specific information concerning the manufacture of both iron and steel rails in that year. In 1877 the production of iron rails amounted to 332,540 net tons, which was a great reduction from the product of 1876, which was 467,168 tons. In 1878 this decline was wholly arrested, the production during the year being fully as great as in 1877, and probably a few thousand tons greater. In 1877 the production of Bessemer steel ingots was 560,587 net tons, and the production of Bessemer rails was 432,169 tons. In 1878 the

production of ingots was about 730,000 net tons, and the weight of Bessemer rails produced was about 600,000 net tons. For these statements we have the best authority. Putting the iron and steel rail products of the year together, we have in round numbers a total of 930,000 net tons as the rail product of the year. This product has only once been exceeded in our history, in 1872, when the product reached 1,000,000 net tons. In 1879 we will probably equal even that immense product.

Production of Lake Superior Iron Ore.—The production of iron ore in this celebrated mining field was much greater in 1878 than in 1877. The shipments in 1878 amounted to about 1,125,000 gross tons, against 960,982 tons in 1877. This large production has only once been equaled in the history of the district, in 1873, when the shipments amounted to 1,167,379 tons.

Railroad Construction.—The mileage of new railroads constructed in the United States in 1878 was very much greater than in 1877, and greater than in any other year since 1873. The *Railroad Gazette* publishes detailed statistics for 1878, which show that in that year there were built and track laid upon 2,688 miles of new railroad, against 2,177 miles in 1877, 2,657 miles in 1876, 1,758 miles in 1875, 2,305 miles in 1874, and 4,069 miles in 1873. The recovery in 1878 from the depression in railroad building which existed in 1877 was marked and decided, and we need scarcely add forms one of the most encouraging signs of the day.

Prices.—The only discouraging feature of the iron trade of 1878 is that which relates to prices. In both iron and steel rails there was an improvement, but in pig iron and bar iron there was a decline from the exceptionally low prices of 1877. The following table shows the range of prices throughout the year:

Months.	No. 1 Anthracite Foundry Pig Iron in Phila.	Iron Rails at Works in Pa.	Bessemer Steel Rails at Works in Pa.	Best Refined Bar Iron in Phila.
January	$18 50	$32 50	$41 00	$44 80
February	18 50	32 50	41 50	44 80
March	18 50	32 50	41 50	44 80
April	18 50	32 50	42 00	44 80
May	18 00	32 50	43 50	44 80
June	17 25	32 50	43 00	44 80
July	17 25	33 00	43 50	44 80
August	17 50	33 00	42 50	44 80
September	17 50	33 00	42 50	44 80
October	17 00	33 00	42 50	42 56
November	16 50	33 00	42 00	42 56
December	17 00	33 00	41 00	42 56
Average	$17 67	$32 75	$42 20	$44 24

The decline in the price of pig iron during the year was $1.50 a ton, and on bar iron it was one-tenth of a cent per pound, or $2.24 a ton. Iron and steel rails sold during the year at average prices which were higher than quotations in January.

Conclusion.—The old year, take it all in all, was a more active and a more prosperous year for the American iron trade than either 1876 or 1877. There was an improvement in the demand for all iron and steel products and prices, although not satisfactory, were well maintained except in the case of pig iron. This branch of the trade has had a hard struggle, and many furnaces have been run without profit. The new year opens with the promise of a still more active and more prosperous business for our iron and steel manufacturers than the old year gave to them. Business is in fewer hands and home competition cannot be so desperate as it has been. Foreign competition is for the present not to be dreaded. Prices it is hoped are at last at the lowest point to which they can possibly fall, while the unmistakable and undeniable revival of general prosperity throughout the country gives every assurance of a continuance of the increased demand for iron and steel which characterized the old year.

THE MISSISSIPPI AS A SILT BEARER.

By ROBERT E. McMATH, C. E.

Written for VAN NOSTRAND'S MAGAZINE.

Amongst the topics engaging the interest of the public, perhaps no one is more worthy the attention of the engineering profession than the discussion of matters relating to the Mississippi River, the improvement of its navigation, the restriction of its floods, the fixation of its bed, and the drainage of its border lands.

These topics are important because of many considerations of public economy, but aside from these, the whole subject is of peculiar interest to the Civil Engineer as being a field, which, in the near future, will afford opportunity for the exercise of the highest order of talent, and for the execution of many very important works.

The profession at large is therefore interested, and should carefully examine all facts, theories and plans advanced; for to engineers the nation must look for that dispassionate discussion and formation of opinion which ought to precede the adoption of any scheme. Schemes looking to this field are already in existence, and, however diverse they may be in origin and ostensible purpose, are alike in this, that they essentially are efforts to monopolize the field for private, or corporate, glory and profit.

To facilitate examination of the various schemes which have been, and plans which may be presented, it will be profitable to inquire into the resources of reliable information upon the subject of the great river.

In a professional, or scientific point of view the information is meager; for the reason that the subject is beyond the reach of private investigation, and those made by the General Government have not been systematically kept up, because Congress never has realized the importance of such studies, and has almost invariably refused to grant the necessary appropriations.

In the way of general information much is in existence scattered through the public documents and periodicals, but mostly unavailable, because few have access to complete sets of public documents, and still fewer to files of the numerous periodicals in which fugitive articles have been published. The reports of Lee, Long, Cram, Kearney, and Shreve, are well worthy of collection and publication for the experiences related, the facts recorded, and the opinions given. Material of like value is scattered through reports made by engineers and others connected with the Levee works executed under State or local organizations; but this too is practically unavailable. Of formal reports available to the ordinary professional reader we have, Ellet on the Ohio and Mississippi Rivers, Humphreys and Abbot's Report on the Mississippi River, and the reports of U. S. Engineers at various dates, since the resumption of civil works at the close of the war of the rebellion, in Reports of Chief of Engineers and other official documents.

None of this material is formally scientific except the report of Humphreys and Abbot, and some of the reports of the inspecting engineer of the jetty works at the mouth of the Mississippi; scientific, I mean in the sense, that in addition to statements of conclusions and reasonings, the data upon which they are founded are also given.

Quite an extensive literature is extant, composed of essays and arguments for and against certain projects of improvement. These arguments are, professedly at least, based on actual observation, and personal acquaintance with the river and the problems involved in a proposition for its improvement.

Personal knowledge, however accurate, cannot furnish a foundation upon which to build a demonstration of any general fact. Its province is limited to practical work, either of improvement or investigation.

The discussions referred to have not therefore added appreciably to our knowledge of the subject, because consisting chiefly of assertions, assumptions, and speculations.

Facts in dry detail, recorded as observed without reduction, interpolation, or generalization, are greatly needed that

every man may scrutinize for himself the foundation of opinions, assertions and assumptions.

In this way only may we hope to establish certain general principles upon which practical schemes may be profitably discussed.

Such statements of fact, when accompanied by an exposition of the mode and means of observation, particularly when there is any novelty in either mode or means, are entitled to the respect and confidence given an honest and capable witness. But beyond the range of statements of fact the weight of personal authority ceases, and the man who is unwilling to submit his every opinion and conclusion to the test of unsparing criticism has no business to enter the field of scientific discussion.

It is not the purpose in this paper to attack, or defend, any man's work, theory, or statement of fact, but rather, to bring prominently forward facts which have unaccountably been left unnoticed hitherto, to point out in what respects our present knowledge is deficient, to suggest means by which the deficiency may be remedied, and to draw, so far as the facts presented will warrant, practical conclusions to guide practical conduct, until further investigation shall show a truer and better way.

Without further preface, I introduce a tabulated statement of the areas of a considerable number of cross-sections of the Mississippi, between the Missouri and Ohio Rivers, showing the stage of water, locality and date of section, the actual area found, the width at the date and stages and the mean depth of section. The division of the table headed "Low water," contains the areas, widths, and mean depths that would result from reducing the data of the preceding columns to the low water of 1863, in all sections above and including Horsetail No. 2, and to the low water of 1872 in all sections below Horsetail.

The low water of 1872 reads 1'.87 above the low water of 1863 upon the present iron gauge at the foot of Walnut Street, St. Louis, (as it is commonly described, though really it is near midway between Market and Walnut) and 2,50 upon the gauge at the St. Louis Elevator, which was the one used by the Signal Service Observer in 1872.

TABLE I.

Date.	Locality.	Stage.	Actual.			Low Water.				Remarks.
			Areas.	Width.	Mean Depth.	Areas.	Width.	Mean Depth.	Year.	
			sq. ft.	ft.	ft.	sq. ft.	ft.	ft.		
July 6, 1878	Cahokia Chute....	22.0	39,346	1798	21.88	4,598	902	5.09	1863	by Wm. Popp.
"	Arsenal.....	22.0	38,158	1915	19.92	3 875	830	4.67	"	"
"	Sum of both Chutes	22.0	77,504	3713	20.87	8,473	1732	4.89	"	"
May 17, 1873	Cliff Cave.....	21.8	73,664	3440	21.40	9,016	2570	3.51	1872	by J. D. McKown
May 28, 1872	Carondelet.....	20.0	63,311	2806	22.56	14.839	1480	10.26	1863	by Capt. Allen.
July 23, 1873	Brickey's Mill.....	19.6	54,152	1850	29.30	21,595	1482	14.57	1872	by J. D. McKown
May 6, 1873	Horse Tail Bar, 1..	18.5	61,725	4800	12.86	0	0	0	1863	"
"	" " 2..	18.5	55,211	4250	12.75	157	90	1.74	"	"
Oct. —, 1837	Bissell's Point.....	16.8	53,253	1835	29.02	31,380	1145	27.44	"	by Lt. R. E. Lee.
April 9, 1872	Horse Tail Bar, 1..	16.5	74,536	4800	15.53	3,694	2110	1.75	"	by S. E. McGregory
"	" " 2..	16.5	66,948	4250	15.51	5,429	1890	2.87	"	"
July 13, 1874	*1 Mile above Ohio	14.54	39.508	2475	16.3	7,078	1520	4.67	1872	by J. D. McKown
June 5, 1874	*Philadelphia Point	11.75	42,187	3740	11.20	12,807	2002	6.39	"	"
Nov. 25, 1876	Cahokia Chute....	10 5	24,492	1580	15.50	11,332	1062	10.67	1863	by Wm. Popp.
"	Arsenal.....	10.5	21,515	1898	11.33	4,078	835	4.88	"	"
"	Sum of both Chutes	10.5	46,007	3488	13.19	15,410	1897	8.12	"	"
Aug. 23, 1873	*Above Chester ...	10.25	26,912	1740	15.5	14,986	850	17.0	1872	by J. D. McKown
Oct. —, 1843	Bissell's Point.....	10.06	33,320	1980	16.77	15,324	1500	10.21	1863	by Capt. Cram.
"	Venice Ferry.....	10.06	34,287	2366	14.50	13,091	1848	7.08	"	"
"	Bloody Island, East	10.06	16,071	1373	11.70	5,609	707	7.91	"	"
"	" " West	10.06	25,241	1721	14 66	9,538	1400	6.81	"	"
"	Sum of both Chutes	10.06	41,312	3094	13.35	15,147	2107	7.19	"	"
Sept 10, 1872	Penitentiary Point.	9.08	47,395	2465	19.23	25,552	1784	14.32	1872	by Clem'nt Smith

TABLE I—(*continued*).

Date.	Locality.	Stage.	Actual.			Low Water.				Remarks.
			Areas.	Width.	Mean Depth.	Areas.	Width.	Mean Depth.	Year.	
	Penitentiary Point—		sq. ft.	ft.	ft.	sq. ft.	ft.	ft.		
Sept. 10, 1872	4800′ below.....	9.08	33.572	1572	21.35	21,293	1094	19.46	1872	by Clem'nt Smith
"	Brickey's Mill.....	9.08	37,565	1683	22.32	23,157	1356	17.08	"	"
Sept. 11, 1872	4000′ below"	9.0	44,097	2654	16.61	20,901	2502	8.35	"	"
"	7900′ below "	9.0	40,861	3657	11.17	11,958	2612	4.58	"	"
Sept. 26, 1872	Above Chester....	8.18	30,795	1219	25.26	20,402	1088	18.75	"	"
Sept. 27, 1872	*Chester.....	8.56	37,562	1505	24.96	25,108	1403	17.90	"	"
——— 1843	Bloody Island, East	7.6	14,744	1220	12.08	6,494	980	6.63	1853	by Capt. Cram.
——— "	" " West	7.6	24,020	1750	13.72	11,983	1460	8.21	"	"
——— "	Sum of both Chutes	7.6	38,764	2970	13.05	18,477	2440	7.08	"	"
Oct. —, 1846	Bloody Island, East	7.6	9,452	680	13.90	4,322	670	6.45	"	by Henry Kayser
"	" " West	7.6	26,445	2000	13.22	11,321	1980	5.72	"	"
"	Sum of both Chutes	7 6	35,897	2680	13.39	15,643	2650	5.90	"	"
Sept. —, 1847	Bloody Island, East	7.6	12,966	1110	11.68	5,708	800	7.13	"	"
"	" " West	7.6	26,181	1700	15.40	13,337	1680	7 94	"	"
"	Sum of both Chutes	7.6	39,147	2810	13.93	19,045	2480	7.68	"	"
Oct. 16, 1872	*Vancil's Landing.	7.4	34,042	3904	8.72	6,132	3710	1.65	1872	by Clem'nt Smith
Oct. 17, 1872	*MoccasinSpr'gsF'y	7.4	29,124	3171	9.18	11,743	1796	6.54	"	"
"	*2100′ below "	7.4	32,498	3031	17.22	11,634	2168	5.37	"	"
"	*5100′ " "	7.4	32,350	3394	9.62	19,541	806	24.24	"	"
Dec. 9, 1870	Bischoff's Dike...	7.3	29,940	1430	20.94	19,520	1310	14.90	1863	by S. E. McGregory
Oct. 16, 1877	" ...	7.0	18,909	1510	12.52	7,677	1388	5.53	"	by Wm. Popp.
Dec. —, 1844	Venice Ferry.....	7.0	44,447	2376	18.71	27,815	2200	12.64	"	by Capt. Cram.
"	Kerr's F'd to Gingras	7.0	33,072	2794	11.81	13,822	2662	5.19	"	"
"	Market Street.....	7.0	53,820	3740	14.39	29,180	3300	8.84	"	"
Oct. 23, 1873	*Cape Girardeau ..	6.9	20,756	1780	11.66	10,565	1172	9.01	1872	by J. D. McKown
Sept. 30, 1838	Venice Ferry.....	6.8	28,910	2587	11.17	15,630	1320	11 81	1863	by Capt. R. E. Lee
"	Market Street.....	6.8	30,600	3511	8.71	9,078	2719	3.34	"	"
Nov. 14, 1872	*Sister Island	6.43	24,939	1866	13 37	12,660	1717	7.37	1872	by J. D. McKown
Oct. 31, 1872	*Foot Grand Chain	6.36	29,715	1953	15.21	18,729	1630	11.49	"	"
Nov. 20, 1872	*Dickey's Island..	6.1	17,168	1320	13.00	10,617	1047	10.14	"	"
— —— 1842	Venice Ferry.....	6.0	32,790	2684	12.22	17,016	2244	7.58	1863	by Henry Kayser
Dec. 4, 1874	Cahokia Chute....	6.0	10,678	997	10.71	6,292	464	13.56	1863	by J. D. McKown
Dec. 5, 1874	Arsenal	6.0	15,917	1828	8.71	6,995	1146	6.10	"	"
"	Sum of both Chutes	6.0	26,595	2825	9.41	13,287	1610	8.25	"	"
Sept. —, 1839	Kerr's F'd to Gingras	4.8	36,732	3260	11.27	22,635	2614	8.66	"	by Capt. R. E. Lee
"	Duncan's Island...	4.8	28,248	2191	12.89	18,178	2006	9.06	"	"
"	" " " ...	4.8	29,541	1827	16.17	20,952	1756	11.92	"	"
Jan. 21, 1861	Plum St., St. Louis	3.25	24,779	1875	13.21	18,832	1785	10 55	"	by C'y Eng'r St. Louis
"	Cedar "	3.25	24,814	1930	12 85	18,915	1700	11.12	"	"
"	Mulberry "	3.25	24,764	1920	12.80	18,833	1730	10.88	"	"
"	Lombard "	3.25	25,205	1975	12.76	19,137	1750	10.93	"	"
"	Hazel "	3.25	26,995	1980	13.63	20,797	1835	11.33	"	"
"	Sycamore "	3.25	23,901	1990	12.01	17,645	1860	9.50	"	"
"	Convent "	3.25	24,641	1985	12.42	18,411	1850	9.95	"	"
Feb. 9, 1861	Carroll "	2.27	20,775	1765	11.77	17,294	1400	12.35	"	"
"	Soulard "	2.27	22,289	1865	11.94	18,560	1525	12 17	"	"
Feb. 7, 1861	Rutger "	2.19	21,106	1905	11 08	17,307	1550	11.17	"	"
"	Miller "	2.19	20,726	1680	12.34	17,371	1370	12.66	"	"
"	Barry "	2.19	21,760	1770	12.30	18,092	1565	11.56	"	"
"	Marien "	2.19	20,800	1715	12.13	17,308	1460	11.85	"	"
——— 1865	Biddle "	Soundings made through ice, at a stage below 2′.0.				20,711	1779	11.64	"	"
——— "	Wash "					23,594	1660	14.21	"	"
——— "	Cherry "					22,489	1690	13 31	"	"
——— "	Green "					22,799	1655	13.80	"	"
——— "	Vine "					29,285	1850	15.83	"	"
——— "	Olive "					25,385	1635	15.52	"	"
——— "	Chesnut "					23,995	1580	15.19	"	"
——— "	Walnut "					20,010	1620	12.35	"	"
——— "	Myrtle "					20,973	1778	11.70	"	"
——— "	Lombord "					20,677	1590	13.00	"	"
——— "	Miller "					20,845	1645	12.67	"	"
——— "	Lesperance "					19,605	1440	13.61	"	"

The above table is arranged, in order, according to the stage of water when the section was measured. An arrangement which is in this case allowable, for the contributions by tributaries, except the Maramec and Kaskaskia, are too small to affect the areas, and these excepted tributaries would not noticeably influence the areas when at a low stage. All the sections which are reduced to the low water of 1863 are located above both these main tributaries, also the Cliff Cave section, and those marked with a * are below the Kaskaskia and therefore include both.

A careful examination of the column of actual areas, after making allowance for the ordinary causes of variation, will suggest that the area does not diminish, when the river approaches a low stage, so rapidly as would naturally be expected.

See that part of the table covering stages ranging below 6½ feet and compare with areas at stages between 6½ and 12 feet.

With our suspicions awakened by the study of the actual areas, we turn to the column of reduced areas at low water, and comparing these, as deduced from high and low stages, it is immediately apparent, that the areas deduced from low stage sections are much larger than those obtained from high stage sections. The suspicion then becomes certainty that important changes occur at the bottom of the river, which in some way accompany the change of stage.

While a general study of the whole list leads to the above conclusion, a particular comparison of the columns of areas and widths of the low water sections shows that the areas also increase, within limits, as the width diminishes. This suggestion is chiefly based upon the sections made at St. Louis in 1861 and 1865, and is confirmed by the sections at Carondelet, Brickey's Mill, and others made at the higher stages. It will be noticed that, in nearly every case in which the deduced low water area exceeds 15000 square feet the low water width is 1600 feet or less, or else that the differences of width at the higher and extreme low stage are not great.

Restating our conclusions in other terms we have learned, that while the bed of the river ordinarily rises and falls in some unknown relation to the rise and decline of stage, there is an adjustment of widths and form of section at high and low stages by which this change can be reduced to a minimum, if not prevented altogether.

At this point it is advisable to introduce another tabular statement, derived from the preceding, by selecting those sections which have for any reason been repeated, and comparing the areas found by these repeated measurements. In this table the geographical order of arrangement is followed.

(*See Table on following page.*)

At a first glance, this table may seem inconsistent with the conclusion stated above, but if the sections at Arsenal and Cahokia chutes, Horsetail and Brickeys, which embrace shorter periods than most of the others, are examined, they will be found to abundantly confirm it. The other cases only point to the fact that we have not yet taken notice of all the elements of the problem; in other words, our statement is incomplete.

Strictly speaking, our hypothesis requires comparison of sections taken at short intervals, so that each low water area may be compared with those deduced from the preceding and succeeding high waters; indeed, the effect of each wave of rise and fall should be studied.

Obviously, the rise and fall of the bottom, which we have noted, is connected with the fact that our river is a silt-bearer, which is excessively turbid in times of flood, and only moderately so at low stages. We may, therefore, suspect that these changes at the bottom depend upon the variations in the amount of silt carried by the river at different stages and seasons. Hence we infer: *First*, that after a flood the removal, by the less turbid waters of a declining stage, of the deposit made in the bed during the flood, is a work in which time is an important element; consequently, that a rapid decline from a high stage is unfavorable to the clearing of the river bed. *Second*, that in floods of equal height, coming from different sources, or from the same source at different seasons, the amount of deposit will differ, if the tributaries bring in dissimilar quantities or qualities of material.

Since the Missouri water was found, by the engineers of the St. Louis water-

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			Areas.	Width	Mean depth.	Areas.	Width	Mean depth.
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" , 1843.	"	10.06	33,220	1980	16.67	15,324	1500	10.21
Dec. 9, 1870.	Bischoff's Dike.....	7.3	29,940	1430	20.94	19,520	1310	14.90
Oct. 16, 1877.	"	7.0	18,909	1510	12.52	7,677	1388	5.53
Oct. —, 1843.	Venice Ferry..... ..	10.6	34,287	2366	14.50	13,091	1848	7.08
Dec. —, 1844.	"	7.0	44,447	2376	18.71	27,815	2200	12.64
Sept. 30, 1838.	"	6.8	28,910	2587	11.17	15,630	1320	11.81
——— 1842.	"	6.0	32,790	2684	12.22	17,016	2244	7.58
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July 6, 1878.	Both Chutes	22.0	77,504	3713	20.87	8,473	1732	4.89
Nov. 25, 1876.	"	10.5	46,007	3488	13.19	15,410	1897	8 12
May 6, 1873.	Horsetail Bar, No. 1.....	18.5	61,725	4800	12.86	0	0	0
April 9, 1872.	" "	16.5	74,536	4800	15.53	3,694	2110	1.75
May 6, 1873.	Horsetail Bar, No. 2.....	18.5	55,211	4250	12.75	157	90	1.74
April 9, 1872.	" "	16.5	66,948	4250	15.51	5,429	1890	2.87
July 23, 1873.	Brickey's Mill.....	19.6	54,152	1850	29.3	21,595	1482	14.57
Sept. 10, 1872.	"	9.08	37,565	1683	22.32	23,157	1356	17.08

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The table proves that the bottom changes attending variation of stage, which we have found to be a prominent physical fact in the section of river between the Missouri and Ohio rivers, have a more obscure but real existence throughout the Lower Mississippi. And the well-known fact that the depths upon the bars, at the mouths of the several passes, is greater when the river is at a low stage than during floods demonstrates that the selfsame phenomenon extends to the very mouth of the river.

Interrupting for a time our prescribed course, the attention of those who have been interested in the discussion as to the cause of reported shoalings below Bonnet Carre crevasse and below Cubit's Pass and the Jump, is called to the possibility that the facts upon which the discussion is based could as readily be found, if sought for at the proper time of year, above as below the crevasses or outlets which, by some, are charged with having produced the reported shoalings. Until it is conclusively proven, by soundings repeated at various times, and at different stages of water, particularly low waters, that the reputed shoalings are permanent, and not those attendant upon a high river, arguments based thereon can have no legitimate place in a discussion of the merits of outlets.

Fallacies arising from honest misinterpretation of facts are as misleading as those which have a purely speculative, or even feigned origin. Prof. Forshey, in the paper previously quoted, says (page 25): "It has 180,875 square feet of cross-section, being 2735 feet wide at high water and 2152 feet at low water. It has increased 13,646 feet in section in 23 years." The latter sentence certainly contains one of these honest misinterpretations which might lead to serious results if carried into the category of unquestioned facts.

Returning to our special topic, we may receive as proven by the testimony of a considerable number of sections taken, out of a list of nearly three hundred, so as to cover:

First. The river passing out of a narrow gorge into a wide expanse; at Bissell's Point, Venice Ferry and Kerr's Island to Gingras. (The conditions just mentioned existed when the sections were made, but not now.)

Second. An extended narrow channel of nearly uniform width; in front of the city of St. Louis.

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These various conditions fairly represent the variations to which the river is subject. And a phenomenon which is found to occur under all these conditions may safely be received as a general fact, resulting from an ever-acting cause and governed by determinate law.

Knowing the existence of the general fact, we now may consider how to pass from this general to detailed knowledge of how the results are brought about, and thus to the law of the changes noted.

I intend to remark chiefly upon modes of fluid operations. From what has been said of the value of hydrography and hydrometry, as usually understood, when applied to silt-bearing streams, it is obvious that improved methods must commence with the field surveys.

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and source of waters are elements in the variations of the bottom phenomena, frequent repetitions of some parts of the field work will be of prime importance.

It will be readily admitted, that none of the following observations and determinations can properly be omitted, if the purpose be a thorough study of a silt-bearing stream, and doubtless it will be found necessary to add to the list:

First. Stage of water in the main river and important tributaries observed simultaneously. The relative elevations and distances between gauges should be known.

Second. Shore line changes, whether of main banks or bar outlines.

Third. Repeated located soundings upon fixed lines.

Fourth. Cross sections plotted and areas calculated.

Fifth. Lateral movement of cross section determined.

(*a*) With reference to center of figure;

(*b*) With reference to center of moment of stream.

Sixth. Determination of bottom velocities.

Seventh. Determination of mean velocities.

Eighth. Determination of volumes at all stages.

Ninth. Determination of quantity and quality of material in suspension at various depths.

Tenth. Determination of quantity and quality of material drifted along the bottom.

The above observations should be continued so as to include two low water periods and the intermediate high water, in other words, about fifteen months; beyond that time more or less of the list could be dropped.

Concerning practical methods, it is probably not necessary to say anything so far as the first five items of the list are concerned, for the methods in ordinary use will suffice until improved.

Satisfactory methods of velocity measurement, and for the determination of the movement of drifted matter on the bottom are yet to be invented. It is, therefore, well to develop the conditions of the problem in each of these cases.

So far as discharge measurements are concerned, it is now conceded that the velocity at a depth, somewhere between mid and two-thirds depth, bears a fixed ratio to the mean velocity in a vertical plane, and that between these limits the velocity varies but slightly; therefore, since for the purpose we now have in view absolute discharges are not necessary, if we secure proportional accuracy, it will suffice, if double floats are used, (observing the requisites of good double floats as stated by Gen. Ellis, See part 2, Report of Chief of Engineers U. S. A. for 1875, page 306), to allow a length of suspending cord equal to two-thirds of the depth, and to rest upon the probability that, whatever may be the uncertainty as to the exact position of the lower float, it will not be far from the mean of the several depths given by the different authorities, as the proper position of such a float.

At times, when the depths are varying rapidly by changes at bottom and surface, as is the case when the river is rising or falling rapidly, it is evident that an approximate discharge, determined by soundings and floats observed within the space of a few hours, may be much more accurate than a discharge obtained by a much better method, but by observations occupying a considerable length of time. For our discharges then we may desire, but do not need, better results than can be obtained by double floats.

The controverted question concerning long and short bases for float observations need not be considered with reference to determination of positions. The length of base should be decided with reference to uniformity in cross section, both in area and form, and parallelism of currents, on one hand, and the errors of time interval, as affecting the observed velocity, on the other; the first calling for a short, the latter for a long base.

To determine the position of floats the following method is suggested, and claimed to have the following merits when applied to large and rapid rivers:

1. As great accuracy in position as is practicable by theodolite angles measured from a short base; and a truer determination of the direction and consequent length of the float's path.

2. It simplifies the observations, and lessens the duties of observers, and thus removes all excuses for missed floats, or approximations.

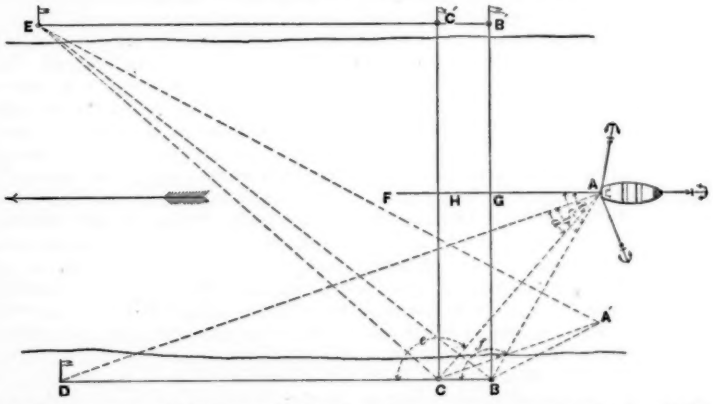
Referring to the accompanying diagram, B, C is the base of observation, which may be taken of any desired length. B and C are each occupied by an instrument. B′ and C′ are range flags; D and E are flags at known points; A is an anchored boat, and AF is the path of any float. An observer is stationed in the stern of the boat with a sextant. From angles *a* and *b*, measured by the sextant, and angles *e* and *f*, measured at C and B, the position of A is approximately fixed, the uncertainty arising from the swing of the boat at its anchor. This being done, B and C fix their cross hairs on B′ and C′, after which a trial float should be run. B and C are to find the float in the vertical range of their telescopes and clamp them, so as to make the path of the trial float central to their field of view. B and C, from this on, have only to watch for the passage of floats, note and record the time of passage, and to let a hand signal fall at the instant the float passes their cross hairs.

A has supervision of starting the floats and watches, through his sextant, by direct vision, the float, and keeps B, by reflection, coincident with it. When B's signal falls (seen in the mirror), A reads angle *c*; then, bringing C into reflection, he watches for C's signal, and when he sees it reads angle *d*. By intersections which may be very readily plotted, we have G and H, giving, with A, three points in the path of the float. If, by any mishap, either angle *c* or *d* is missed, the path may, without serious error, be assumed to be a straight line through A and the position determined. The only error of this system arises from the motion of A, which may be limited at will by using additional anchors.

For positions of A nearer the base than mid-river, as A′, angles for its position may be measured from E instead of D, as indicated.

By this system, but one observer is required to be specially skillful in handling instruments, and his angles are within the range of ordinary tangent screws, and he is therefore little liable to lose an angle. The manual dexterity required to handle the sextant is not difficult of acquirement, and should indeed be part of a hydrographer's regular education.

A time-keeper should be stationed with B or C unless a system of electric signals with chronograph is used.



The determination of bottom velocities is one of the unsolved problems of hydraulics, whose solution, while of special importance because of its relation to the subject of the transport of sediment, is also important to the general subject of hydraulics because involving the discovery of all the now unknown relations existing between velocities at different depths.

The degree of perfection attained in chronographic records by electricity, and in the means of measuring currents by meters, justifies one in saying that the facilities for the solution of the problem of the vertical curve of velocities are now such as to demand further effort.

It may properly be assumed that the determination of bottom velocity depends upon the possibility of determin-

ing the vertical curve of velocities; that is (since it cannot be directly observed in a muddy stream), if the law of the vertical curve is known, the bottom velocity would be ascertained by completing the curve when three or more points in it are known. Previous experimenters have entertained this view, and many efforts to determine the law of the curve have been made, but the law has remained undiscovered.

If we now consider the Horsetail Bar sections Nos. 1 and 2, which were 1930 feet apart (and the same facts extended still further), we note the entire disappearance of the area when reduced to low water, and, in fact, the deepest sounding in No. 1 was 17 feet at the 18'.5 stage of May 6th, 1873, and the same in No. 2, except a single depth of 22 feet, close to the west shore, near the face of a rock ledge, which sounding allows a small remnant of the section to appear in the list.

These sections, made in May, cannot be supposed to represent the maximum filling, for this maximum would naturally occur at the June rise, which that year was well marked, lasting some six weeks. Indeed, we know that, at this locality, there has been as little as six feet of water in the main channel when the gauge readings showed the river at a 14-foot stage. The soundings of May, 1873, were made after a rise to 25.45, culminating April 11th, which rise probably was due to the Upper Mississippi, or to local rains. The sections of April 9th, 1872, were made after a short and quick rise, beginning March 29th and culminating, April 3d, at 17.65, which is almost conclusive proof that it was caused by local rains affecting only the nearer tributaries, conditions favorable to a moderate filling of the low-water channel.

The conclusions of our analysis are then borne out by the facts, so far as they are on record. It would be more satisfactory if we could trace the exact source of the several waves, but the data are not at hand, and will not be until gauge records are kept at points on every important tributary to the Mississippi, far enough from the main stream to be beyond the influence of back-water, and below the main affluents of the tributary.

The facts and analysis are given to lay before those not familiar with the river itself proof of one cardinal fact of great practical importance, which either has altogether escaped the attention, or has not been duly appreciated by those who have undertaken to explain the physics of this river. Persons familiar with the river will possibly recognize in the statements which have been made little, if anything, more than is implied in the maxims, current among boatmen, about the channel flattening when the river rises and cutting out as it falls; also, that a rapid decline from a high stage is followed by a season of bad navigation through shifting channels; also, that a spurt of a rise, coming after the cutting-out process has developed, is considered undesirable; and, again, such statements as the following: "A rise of a foot, coming from the Missouri, adds nothing to the channel depth out to Cairo, but a rise coming out of the Illinois will add materially to the channel depth, even though it does not increase the gauge readings at St. Louis." These results of the observation of practical men confirm the deductions from a study of the cross sections, which we now restate.

A deposit of silt in the bed of the river is made when the turbidity of the stream passes a certain limit, and is removed when the turbidity falls below that limit. These conditions usually attend—the one a rising and the other a falling river. The amount of deposit depends also upon the sources from which the flood waters come and the subsequent removal upon the time occupied by the decline.

I would include in the term turbidity all material conveyed by the stream, whether borne in suspension, swept along the bottom, or moving as a semi-fluid mass along the bed with its own motion as a semi-fluid.

Without here indicating an opinion as to the actual existence of all these modes of transport of sedimentary matter, and particularly the latter, I will introduce the testimony of witnesses as follows:

The late General R. E. Lee was, in 1838, in charge of the improvement of the Des Moines Rapids, by excavating a channel. In his report of October 24th, 1838, he says: "Unless it (the rock loosened by blasts) is removed, the effect of the second course of blasts is much diminished, and the current of

sand, constantly drifting over the bed of the river, soon fills up the crevices and renders reblasting necessary." The context shows that the operations were limited to low stages affording wading depths; therefore the comparative transparency of the water, at that locality and stage, favors the supposition that Lee's statement of a "current of sand constantly drifting over the bed of the river" was founded upon direct observation. If the current of sand existed at low water, it certainly did at high.

It is related, upon the authority of the chief engineer of the bridge over the Missouri at St. Charles, that during the sinking of one of the caissons certain fender piles, which had been driven above the caisson, disappeared, and when the caisson reached the bed rock the lost piles were found lying on the rock under the caisson. What depth of sand overlaid the rock was not stated, but it was considerable.

Again, in the report of the chief engineer of the Illinois and St. Louis Bridge of June, 1868, page 21, it is said: "I had occasion to examine the bottom of the Mississippi below Cairo during the flood of 1851, and at 65 feet below the surface I found the bed of the river, for at least three feet in depth, a moving mass, and so unstable that, in endeavoring to find footing on it beneath the bell, my feet penetrated through it until I could feel, although standing erect, the sand rushing past my hands, driven by a current apparently as rapid as that at the surface. I could discover the sand in motion at least two feet below the surface of the bottom, and moving with a velocity diminishing in proportion to the depth at which I thrust my hands into it." In this very explicit and circumstantially detailed statement we are assured that the bottom was moving in time of flood, and under water of very considerable depth—65 feet.

Professor C. G. Forshey, in a paper entitled "The Physics of the Gulf of Mexico, and of its chief affluent the Mississippi River" read at the Nashville meeting of the American Association for the Advancement of Science in August, 1877, Page 18 of pamphlet copy says: "It was ascertained early, say within the first month of observation, that there was some other undiscovered method of explaining the amount thrown out on the sand-bars, in the crevasses of the levees and banks, and carried to the bars of the river at its mouths.

"A single experiment made by the writer of these pages brought to light the secret.

"The matter was borne along the bottom of the river, too heavy to be held in suspension. By far the greater amount passed off thus invisible. The amount could not be measured. The experiments were continued. Samples were taken during the whole of two years and the drift matter never was found absent beneath a running current. From the greatest depth on the line of observation and from the gently sloping shore of the bar on the other side, the parcels were taken.

"After much reflection and desire to make the estimate reliable, I have concluded to estimate the amount of drift matter, rolled or pushed along the bottom of the river, at three-fourths of the whole quantity by weight."

We have now from four independent sources direct testimony, based on actual observation, all agreeing as to the one thing under investigation, namely, that very important movements are in progress at the bottom of the river, and we may add, at all stages of water, at all depths, and at all parts of the river, for our evidence is drawn from both of the great rivers whose union makes the true Mississippi, from the main stream after receiving its other great tributary, the Ohio, and from the deep and comparatively stable section at its approach to the Gulf. A fact, which is invariably found under such varying conditions, must be a general one. Of the detail of this general fact no one can pretend to have an adequate knowledge, and yet it must be admitted that such accurate knowledge is of the highest importance, both in a scientific and practical point of view.

Regarded from the scientific standpoint, since we have proven that the flow of sand is inseparable from the flow of water (an idea which is in fact involved in the most ordinary conception of a silt-bearing stream) it must follow, that the hydraulics of such a stream are more complicated than of rivers of clear water traversing stable beds.

TABLE III.

Locality.	No. of Section, H and A.	High Water.								Low	
		Areas. Sq. ft.	Difference. Sq. ft.	Per Cent. of Change.	r or Hyd. Mean Depth.	Difference. ft.	Width.	Difference.	Center of Moment about base.	Areas. Sq. ft.	Difference.
Columbus.	1	161,419	—	+.018	69.94	—	2260	—	1061.5	64,274	—
	2	164,350	+2931	—	71.61	+1.67	2260	0	1086.4	65,585	+ 1311
	3	164,568	—	+.066	69.26	—	2320	—	1087.0	64,286	—
	4	175,345	+10777	—	74.61	+5.35	2320	0	1119.8	72,604	+ 8318
Vicksburg.	13	178,648	—	—.001	58.15	—	3056	—	1776.9	55,399	—
	14	178,450	— 198	—	57.93	—0.22	3050	—6	1737.2	55,800	+ 401
	15	177,392	—	+.025	57.60	—	3040	—	1732.9	55,003	—
	16	181,800	+4408	—	59.23	+1.63	3034	—6	1725.7	59,865	+ 4862
Carrollton, La.	48	215,643	—	+.0008	70.12	—	3035	—	1561.5	172,330	—
	49	215,806	+ 163	—	70 52	+0.40	3035	0	1556.3	172,493	+ 163
	55	235,450	—	—.039	87.69	—	2605	—	1004.0	197.837	—
	56	226,267	—9183	—	84.90	—2.79	2605	0	936.5	188,805	— 9032
	56	233,193	—2257	—	—	—	—	—	—	—	—
	57	219,381	—	+.031	87.05	—	2460	—	923.6	184,731	—
	58	227,458	+8077	—	90.62	+3.57	2460	0	941.5	192,808	+ 8077
	58	226,877	+7496	—	—	—	—	—	—	—	—
	62	218,393	—	+.081	73.78	—	2930	—	1181.3	178,980	—
	63	236,069	17676	—	79.75	+5.97	2930	0	1221.1	196,656	+17676
	64	208,141	—	+.063	74.60	—	2760	—	1080.5	170,041	—
	65	222,036	+13895	—	79.30	+4.70	2760	0	1039.8	183,673	13632
	68	190,080	—	+.101	72.55	—	2575	—	984.9	154,630	—
	69	209,241	+19161	—	80.01	+7.46	2575	0	1052.6	172,941	+ 8311
	69	229,325	—	—	—	—	—	—	—	—	—
	70	190,538	—	+.110	73.28	—	2550	—	959.6	154,990	—
	71	211,530	+20992	—	81.67	+8.39	2550	0	1015.6	175,305	+20315
	72	191,320	—	+.102	74.01	—	2530	—	952.4	156,410	—
	73	210,937	+19617	—	81.61	+7.60	2530	0	1027 2	176,012	+19602
	74	185,312	—	+.087	71.24	—	2525	—	968.5	149,361	+
	75	201,446	+16134	—	78.70	+6.46	2525	0	998.2	165,495	+16134
	76	185,338	—	—	—	—	2705	—	—	150,030	—
	77	171,278	—	+.104	66.77	—	2530	—	994.5	135,327	—
	78	189,128	+17850	—	73 59	+6.82	2530	0	1014.5	153,177	+17850
	78	194,945	+23667	—	—	—	—	—	—	—	—
	79	173,014	—	—	—	—	2670	—	—	138,200	—
	80	170,096	—	+.095	66.05	—	2540	—	1010.5	135,000	—
	81	186,333	+16237	—	72.32	+6.27	2540	0	1028.3	150,520	+14520
	81	188,160	+18064	—	—	—	—	—	—	—	—
	83	192,746	—	—.040	72.87	—	2600	0	995.1	156,033	—
	84	185,092	—7654	—	69.94	—2.93	2600	0	971.1	148,342	— 7691
	85	204,627	—	—.001	75.93	—	2650	0	1036.9	167,164	—
	86	204,348	— 279	—	75.97	+0.04	2650	0	1046.8	166,885	— 279
	88	184,574	—	—	—	—	2925	—	—	145,200	—
	88	198,302	—	—	—	—	—	—	—	—	—
	89	189,268	—	—	—	—	2950	—	—	149,010	—
	89	197,914	—	—	—	—	—	—	—	—	—
	91	220,246	—	—	—	—	2780	—	—	180,000	—
	91	188,075	—	—	—	—	—	—	—	—	—
	92	202,228	—	—	—	—	2600	—	—	164,115	—
	92	205,117	—	—	—	—	—	—	—	—	—

TABLE III—(*continued*).

Water.									
Per Cent. of Change.	r or Hyd. Mean Depth.	Diff.	Width.	Diff.	Center of Moment about base.	No. of Soundings in Section.	Month.	Year.	Remarks.
+0.02	33.47	—	1905	—	1017.07	46	Dec.	1857	Rising Rapidly.
—	31.37	−2.10	1955	+50	1059.60	73	Oct.	1858	Falling.
+.129	31.60	—	2020	—	1039.1	41	Dec.	1857	Rising Rapidly.
—	35.76	+4.16	2025	+ 5	1104.2	58	Oct.	1858	Falling.
+ .007	23.59	—	2340	—	1978.2	28	Feb.	1858	" mean stage.
—	23.27	−9.32	2385	+45	1923.6	48	Sept.	1858	" " lower.
+ .088	23.45	—	2335	—	1907.0	36	Feb.	1858	" "
—	24.94	+1.49	2395	+60	1919.7	36	Sept.	1858	" " "
+.0009	62.10	—	2740	—	1554.0	31	June	1851	High stage.
—	62.50	+0.40	2740	0	1563.3	23	Nov	1851	Low "
— .046	79.61	—	2410	—	948.8	24	June	1851	High "
—	77.06	— 255	2390	20	875.1	23	Nov.	1851	Low "
—	—	—	—	—	—	—	—	1872	———
+ .044	83.21	—	2160	—	901.8	23	June	1851	High "
—	87.24	+4.03	2160	0	897.3	24	Nov.	1851	Low "
—	—	—	—	—	—	—	—	1872	———
+ .098	75.20	—	2325	—	1125.8	21	June	1851	High "
—	84.04	+8.84	2325	0	1214 0	21	Nov.	1851	Low "
+ 080	72.51	—	2320	—	1036.4	21	June	1851	High "
—	76.85	+4.34	2355	+35	1053.5	22	Nov.	1851	Low "
+ .118	67.67	—	2245	—	915.0	43	June	1851	High "
—	75.03	+7.36	2265	+20	1003.9	36	Nov.	1851	Low "
—	—	—	—	—	—	—	—	1872	———
+ .131	66 95	—	2275	—	902.6	43	Feb.	1851	Rising after fall.
—	75.56	+8.61	2280	+ 5	971.8	38	Sept.	1851	Falling.
+ .125	69.66	—	2240	—	884.8	45	Feb.	1851	Rising after fall.
—	76.03	+6.37	2260	+20	990.1	37	Sept.	1851	Falling.
+ .108	63.42	—	2223	—	903.0	45	Feb.	1851	Rising after fall.
—	72.91	+9.49	2238	+ 15	953.4	31	Sept.	1851	Falling.
—	—	—	2260	—	—	—	Feb.	1859	Mean stage.
+ .132	60.41	—	2219	—	938.2	44	Feb.	1851	Rising after fall.
—	66.75	+6.34	2259	+ 40	974.7	30	Sept.	1851	Falling.
—	—	—	—	—	—	—	—	1872	———
—	—	—	2220	—	—	—	Feb.	1859	Mean stage.
+ .107	59.60	—	2230	—	949.2	44	Feb.	1851	Rising after fall.
—	65.86	+6.26	2235	+ 5	988.7	34	Sept.	1851	Falling.
—	—	—	—	—	—	—	—	1872	———
— .049	66.82	—	2295	—	950.7	27	June	1851	High stage falling.
—	63.40	−3.42	2300	+ 5	902.2	24	Dec.	1851	Low "
— .002	69.94	—	2345	—	975.0	23	June	1851	High " "
—	69.97	+0.03	2345	0	98.19	24	Dec.	1851	Low "
—	—	—	2500	—	—	—	June	1851	High " "
—	—	—	—	—	—	—	—	1872	———
—	—	—	2595	—	—	—	June	1851	High " "
—	—	—	—	—	—	—	—	1872	———
—	—	—	2460	—	—	—	June	1851	High " "
—	—	—	—	—	—	—	—	1872	———
—	—	—	2355	—	—	—	June	1851	High " "
—	—	—	—	—	—	—	—	1872	———

Therefore, observations of a special character are required to detect the complex laws of a sediment-bearing stream. Take so simple a matter as a hydrographic survey, for example: these bottom changes forbid—*First:* An attempt at reduction of soundings to a standard low-water stage, as is customary in hydrographic work. *Second:* They render it impracticable to join surveys made at different dates. *Third:* No direct comparisons can be made between different surveys covering the same ground, if such surveys are made at different stages of water, or at the same stage after a diverse succession of stages. *Fourth:* It is impossible to distinguish permanent changes of contour from the fluting ones due to the silt movement. No argument can be required to establish these statements. Every one must see that they are inevitable consequences of the fact of bottom changes attending fluctuations of stage.

Passing from hydrography to hydrometry it is equally obvious that the important element of depth is subject to variations from below as well as at the surface when the river rises and falls. Therefore, a continued series of discharge measurements will be subject to great errors, in respect to areas, velocities, and volume, if the ordinary assumption of unchanged bottom be indulged, and the same remark applies to attempted studies of the vertical curve of velocities.

This seems to have escaped the notice alike of hydraulic authors and critics, though ample facts to show the existence of the error have been in print for years.

I have introduced a tabulated statement (pp. 228–9) from Humphreys and Abbot's Appendix C, of remeasured cross-sections, adding thereto a remeasurement in certain cases made in 1872, the latter on the authority of Prof. Forshey's paper already quoted, and to be found on the last page of the pamphlet copies.

The order of arrangement is geographical, and the table shows the same facts as the preceding ones, except that the hydraulic mean depths are substituted for mean depths, in order to specially direct attention to the influence of bottom changes upon the elements usually entering into hydraulic formulas. The columns headed "center of moment about base" were made out partly with the hope of being able to show a measure of the lateral vibration of the river section, between high and low stages, in passing around a well-defined bend, as at Carrollton; and, in other cases, to show the extent of lateral movement, in order that it might be seen that no marked shifting of the bed occurred between the dates of measurement, which might account for the change of areas. The first purpose failed, because the sections are not real—that is, actually measured at high and low stages—consequently the demonstration of a very interesting fact must be deferred until real data are available.

The absence of the specific dates when the measurements were made detracts from the value of the facts here recorded. The general state of the gauge during the month in which the measurements were made is given in the column of remarks.

Since none of the areas given were actually measured, but, as they stand in the table, are really predicted areas based upon data, obtained at some intermediate but not definitely-known stage, therefore areas predicted from sections measured at comparatively high stages will be smaller than those predicted from measurements at low stages, if the theory advanced in the foregoing pages is correct.

If the reader has access to the work of Humphreys and Abbot, it will be profitable to turn to the gauge records of Appendix B for localities and years named when examining the foregoing table.

Although several writers have brought forward formulas derived from the data in their possession the result is nothing but discord, because the points on the curves were not determined by simultaneous observations.*

* Since the paragraph, giving credit for the legitimate derivation of formulas from the data by various writers on hydraulics, was written, grave doubt has arisen as to the legitimacy of instituting comparisons of vertical curves, taken at various localities and stages, by dividing the observed curves into tenths of the total depth and plotting to a uniform scale the co-ordinates thus obtained; and the further reduction to a mean velocity of unity by dividing all the ordinates in each curve by its mean velocity. These steps are founded upon an assumption, which may or may not be true, that larger and smaller streams are proportionally affected as to their several parts or elements. Taking an extreme case for example, a rivulet flowing under certain conditions is transformed by heavy raintall into a torrent; can it be supposed that the conditions of flow have simply expanded symmetrically? and yet this is really the assumption when the procedure criticized is applied to observations at various stages of water to bring them into condition for a comparison of things which, for aught we know, may be essentially unlike.

Taking the existence of pulsations or irregularities of currents as an established fact, it will be readily understood, that observations at the several depths at different times, and probably for irregular periods, cannot be expected to give a true mean curve. If this point be admitted, and I think it must be, the anomalous results of all previous observations are accounted for, and the remedy suggested. For, by the use of current meters and a chronographic register, it would be possible to make simultaneous observations at as many points of the vertical as we choose to provide meters. And if the revolutions of the several meters are recorded upon one chronograph, data in convenient and unmistakable form will be available for the determination of the vertical curve at any instant; or, by taking means, by groups, of obtaining a series of true mean curves, which, when extended to cut the bottom, will furnish the only possible solution of the problem of bottom velocity.

Looking at this suggestion from a practical point of view, it is probable that it would require but little more trouble and time, to run a series of meters when the vertical wire required for one is in place than to run that one singly, and less if the single meter be run successively at as many different depths as the series. If I am right in this, the cost of a complete series of simultaneous observations would not be much more than an incomplete system would cost, aside from the first expense for meters and electrical apparatus.

The foregoing suggestions it is hoped may prepare the way for the solution of some of the unsettled questions concerning the flow of streams. I, for one, do not believe that the presence of grit in the water presents any insuperable obstacle to the use of properly constructed meters in the Mississippi, though patient and protracted experiment may be required before that proper construction is realized.

My statement of the deficiency of practical methods included the determination of the quantity and quality of drifted material.

The collection and determination of suspended matter is merely a matter of patient careful detail; and the same may be said of material drifting along a stable bottom.

A trap or dredge similar to to the slip water bottle used on the Challenger expedition can certainly be devised, which will bring the desired specimens to the surface for examination.

But if there is a stream of sand running beneath that of water, the question of detecting and measuring the movements of this under-current is important. It will be remembered that, when speaking of the various modes of transport of sediment, I reserved expression of opinion concerning the weight to be given certain statements which I then quoted.

The statements of Gen. Lee and Prof. Forshey imply no more than the movement of sand and other bodies along and in close proximity to the bottom; but when that bottom is itself composed of sand or other material, readily moved by any disturbing cause, it will be readily understood, that no definite line of division can be drawn between material in motion and that at rest. But the observations incidentally made by means of the caisson and diving bell seem to testify to an extensive movement at the bottom, which in quantity moved would render the movement in suspension trifling in comparison, presenting a question concerning which, we cannot afford to remain in doubt. The evidence quoted is suggestive, but not conclusive; for the presence of so large a body as a caisson or diving bell, would of itself tend to produce the facts observed, by creating a violent disturbance of the local fluid currents, which would react upon the unstable material of the bottom. Some more reliable method of observation must be adopted. Possibly the presence of a diver in armor would not create disturbance enough to vitiate observations made directly as to the principal facts, but exact measurement of the depth and velocity of movement would present more difficulty; which difficulty need not be attacked until the demonstration of the fact makes its solution necessary.

The thorough investigation, suggested in the preceding pages, into the subject of silt movements, is not simply in the interest of the Mississippi, but would contribute to the science of hydraulics in respect to other silt-bearing rivers. Unquestionably every general fact brought

to light will add to the ability of engineers to deal successfully with such streams. But it is not necessary that the application of the very imperfect knowledge we now have to practical experiments should await the result of scientific investigations, for really the two are so intimately related that it would be useless to attempt to separate them Theory must be tested by practice, and practice should continually improve, as theories are perfected.

The general fact, whose existence and influence it is the chief design of this paper to trace, may be summed, in language intended, so far as practicable, to avoid expressing theory as to how or why the phenomena exists, thus—

First. The lower part of the bed of a silt-bearing stream is filled with sedimentary material at high stages, and emptied at low in a manner analogous to that in which the upper part of the same bed is filled and emptied of water.

Second. Unusually narrow parts of the river either receive less than an equal proportion of the deposit, compared with equal areas of wide parts, or else the process of removal of the deposit is more speedy in the one case than in the other, with the probabilities in favor of the latter supposition.

Third. The amount of deposit depends upon the source from which a flood comes and the degree of its subsequent removal upon the time occupied by the decline.

Another class of deposits due to material derived from neighboring caving banks is not included in the above statement; for the period of greatest caving corresponds in most cases with the falling stage. This class of deposits probably has much to do with the condition of navigation, but must be considered separate from those continuous movements which constitute the river a silt-bearer. Caving banks and resulting bars are found in clear streams as well as in the most turbid. These would be local, but the feature we are tracing is found, with rare exceptions, wherever the test is made.

The practical influence of the above-stated facts is very important, for, after a June rise, to a stage of 25 feet or upwards, the natural river does not furnish a continuous navigable channel above the Ohio, 12 feet in depth, at any time after the stage has fallen to 20 feet—that is, the low water channel of the preceding year is entirely obliterated, and, in addition, a fill of eight feet or more in depth, above the plane of low water, is made upon one or more of the bars between the Missouri and Ohio.

It will readily be understood that an improvement, which, following clear-stream precedents, looks only to furnishing a required depth at the lowest stages, may fail to secure even that depth at mean stages, because works designed with a view to low water exclusively will not have the height requisite to secure active influence at higher stages. It has been found by experience that, to secure a depth of 8 feet or more, at all stages, it is necessary that the works should be so designed as to be operative at all stages below 20 feet, which is satisfied by giving dikes and dams an elevation of 13 or 14 feet.

It has been found practicable to build dikes and other works upon these sand foundations by giving them a broad base of brush. Furthermore, it is found that, in the interest of economy, it is desirable to lay these foundations as rapidly as possible, immediately after the culmination of the mid-summer rise, in order to retain the deposits when at their maximum elevation. Practically, operations should commence as soon as the cessation of drift renders it possible to work.

The greater depth at that time is not wholly inimical to success, for, while adding to the difficulty of placing foundations, it affords an area of cross-section, which is so slightly diminished by the foundation courses that no serious increase of current or scour is excited. A dam is now in progress across Cahokia chute, the foundation for which was laid upon a section not materially changed from that given in Table 1, as of the date of July 6th, 1878, and stage 22 feet.

The writer suggested, several years since (in 1868), that, in cases where the reclamation of ground from the river was desired, or the narrowing of the channel to improve the navigation, advantage might be taken of the high water deposits, and their permanence secured by rapidly-constructed barriers against their removal. The same suggestion, with the

addition of the possibility of utilizing, dredging or scraping as a means of aiding navigation, by taking advantage of the high state of the bars early in the fall, is made in the Report of Col. James H. Simpson, on Part of Third Subdivision of the Mississippi Transportation Route, Appendix CC 4 to Report of Chief of Engineers U. S. A. for 1875, page 490.

The fact under consideration may then be treated as a helpful agency in the work of regulating the bed of the stream and improving its navigability; the practical problem being to avail ourselves of the help and avoid the hindrances, which the silt-bearing character of the river renders possible.

Study of the sections given in the tables will certainly justify the position maintained by the writer for several years, that the floods of the Mississippi are not simply the consequence of an extraordinary volume of water, but that they, in no small degree, depend upon the order and intervals at which the rises from the several great tributaries follow each other. The facts that have been presented show how considerable a part of the ordinary bed is occupied by deposited material at ordinary high stages, and strongly suggest the probability of vastly greater deposits in times of great floods.

If the maximum of deposit precedes the maximum of water discharge (as it will if a flood from a silt-bearing tributary, arriving when the main river is at a comparatively low stage, is quickly followed by a flood in the main stream) it is very certain that the flood level will rise much higher for a given volume, than when the clearer tributaries send forth their floods first, and the silt-bearers following drop their burden in a deepened and enlarged bed. So far as the floods are influenced by this cause their control by levees must ever remain uncertain and hazardous. For it is within the range of probabilities that one or more of the lower silt-bearing tributaries will send out a burdened discharge to fill the bed of the main river, and that a flood from other sources following immediately after, or overlapping, the first wave will exceed the ability of the obstructed channel to discharge within banks or levees of any practicable dimensions.

In like manner no certain relief from danger of overflow can be expected to follow from any possible system of outlets. No possible depletion of the lower part of the river can counteract a cause acting along its entire length, which, while most potent near the entrance of the silt-bearing tributaries, at so great a distance as that from the mouth of Red river to Carrollton, is able to cause a variation in a single season of one-ninth of the high water area, as was the case in 1851, at Sections 70 and 71, shown in Table 3. Take another and more striking example. The history of Horsetail bar, a few miles below St. Louis, shows that, in 1876, nearly one-third of the area found between banks, at a thirty foot stage, was filled with sand during the moderate flood of that year.

The point of these statements is, that gauge readings of themselves afford no indication whatever of the volume of the river at either high or low stages. Consequently conclusions reached from a study of gauge records alone will be fallacious, whether the purpose of such study be to establish a grade line for a levee system, or to prove the beneficial effect of certain crevasses, and thence draw an argument in favor of opening artificial outlets.

The facts here presented are an emphatic condemnation of both levees and outlets as anything more than modes of alleviating the evil locally. They do not and cannot fully meet it, and therefore should have no place in plans embracing the whole field and presented or urged as a national work.

In the last volume of this Magazine—September number, 1878, page 223—a plan is suggested, by one who, like the writer, condemns levees and outlets as solutions of the Mississippi problem, which consists in "simply constructing light willow or brush dams during low water on the shoals which are then dry, or nearly so, at the various wide places in the river where the bars always exist."

The files of the office of the Chief of Engineers contain an unpublished paper by the writer, written in 1868, which proposed the same plan. I would now preface a statement, like the above quotation, by asserting that first of all it is neces-

sary in wide places and caving bends to provide by artificial protection a bank, able to resist the abrading and eroding power of the river, against which the concentrated volume of the narrowed river may be safely thrown. For the wide places exist, chiefly, because the banks afford less resistance to erosion than the bed to scour. Clearly the only remedy is to increase the resisting power of the bank. When this is done, permanency of the bank favors permanence in the position of the channel, which in time will be scoured to ample depth for navigation, and the shoal areas may, in many cases, be left to themselves.

Until the progress of the country in population and wealth justifies the radical improvement of the tributaries, the practical problem is to assist the Mississippi to convey to the sea its necessary burden of water and silt. To the professional reader it is not necessary to prove how much traffic is impeded by stoppages and unloading and reloading by the way.

Caving and yielding banks are, in effect, like throwing upon an already overcrowded thoroughfare an immense amount of local traffic. Protection of banks would cut off these local contributions, and the deepened localized channel would have increased ability to carry without interruption its burden of material.

Considered in its broadest aspect there can be little room for dispute that the improvement and maintainance of the navigation, the restriction of floods, the protection against erosions, and the drainage of the bordering lands are so inseparably linked together that the thought of separating them is folly.

Viewed as a whole, it requires no argument to show that the fixation of the bed is the work upon which the successful realization of each and all these desired benefits must depend. It therefore stands logically in the first place in a scheme of improvement, and, in the opinion of the writer, it includes also the realization of all the desired results in no small degree. For the fixation of the bed will lessen the quantity and regulate the movement of sediments to a very considerable extent, and, I have shown, that to these navigation and protection against floods are closely related.

PROGRESS IN PURIFYING SEWAGE BY PRECIPITATION.

From "The Builder."

The object of this communication is to lay before the Congress certain practical and economic results which the writer has arrived at in regard to perfecting the purification of sewage by chemical treatment, and in facilitating the disposal of the sludge derived therefrom. He desires to acknowledge the valuable cooperation he has received from Professor Wanklyn and Mr. J. C. Mellis, C. E.

In considering every system for dealing with sewage, it is desirable that the question should be regarded as a means of enabling sanitary authorities to overcome the great and increasing difficulty in the disposal of their sewage refuse at the least cost, rather than as one for making money out of sewage, which has been too much aimed at, and has thereby thrown back sanitary improvement considerably.

In the course of the writer's experience in advising with reference to precipitation works and systems, he has seen more and more clearly that sulphate of alumina aided by milk of lime proves the most efficient agent to comply with the beforementioned conditions for purifying sewage. This he has now advanced upon, inasmuch as he has discovered that the efficiency of sulphate of alumina is greatly increased by the presence of some protosulphate of iron, which apparently produces results *altogether out of proportion to its chemical equivalent* (the salts of iron being known to chemists as purifiers of foul fluids), the proportions varying with the composition of the sewage, but their combination having a marked sanitary and economic advantage.

* By Mr. Henry Robinson, C. E. Read at the Sanitary Congress, Stafford.

Practical data on a large scale have already been obtained at the sewage precipitation works of the Rivers Purification Association (to whom the writer is engineer) at Coventry and Hertford, where the improved system of working has been adopted with complete financial and sanitary success. The process previously employed at these places was (as is well known with reference to Coventry) crude sulphate of alumina and milk of lime, which gave results admittedly of a highly satisfactory kind. The addition of the salts of iron has, however, led to the production of an excellent effluent, and at a cost even less than previously.

The employment of this combination has also resulted in the tanks and the sludge being deprived of the slight smell hitherto noticeable at times. The economic results attending this compound system of precipitation will be recognized when the writer states that, at the Coventry Sewage Works the purification of the very foul sewage (containing between six and seven grains of chlorine per gallon, and between twelve and thirteen parts per million of albuminoid ammonia, with a large amount of dye from the manufactories) is now accomplished for £1 1*s.* 6*d.* per million gallons.

At Hertford the improved system of working has been applied with equally satisfactory results.

The way to dispose of the sludge from precipitation works has long engaged the attention of the writer. Artificial heat will accomplish it, and several ingenious appliances have been devised for this purpose. The cost, however, is heavy, and the value of the resulting manure is not yet sufficiently established to justify its employment on commercial grounds.

The writer has now adopted the system of removing the water from the sludge by means of filter-presses, and he has devised a simple application to economically effect this.

This press, which is founded on an old and well-known Belgian apparatus for filtering impure liquids, by means of a combination of discs with filtering-cloths on their surface, has been arranged in order to economize labor and time in working, so that the discs open and shut automatically, by which the pressed sludge is removable quickly and without disturbing the filtering-cloths. The fluid sludge is thus converted rapidly and economically into the form of cakes, and the drying can be further carried on to any desired length, either by exposure to the air or otherwise. The difficulty hitherto experienced in disposing of the sludge to farmers for utilization upon land is thus got over, as the writer has found that there is no difficulty in selling the sludge from the process he is referring to, provided it is made portable. The theoretical value assigned to sludge from the writer's process is, according to chemists, 16*s.* 9*d.* a ton if it contains about 50 per cent. of moisture, or 27*s.* a ton if dried down to 15 per cent. of moisture.

It is very well known to those who have traveled in the Alps, that the inhabitants believe that avalanches rarely fall when the sky is overcast, but that they do so, frequently, when the sky grows clear. In winter the monks of St. Bernard always urge travelers not to leave the monastery when the sky is clearing, and many times those who have neglected that advice have fallen victims to their imprudence. M. Dufour, in a paper read before the Paris Academy of Sciences, endeavors to explain the phenomenon by reference to the contraction and decrease of strength of snow and ice under decrease of temperature. "In cold weather," he says, "when the sky clears off the temperature falls, especially just before sunrise, and then the filaments of ice which retain the snow on the slopes of the mountains contract and snap, the mass begins to slide, and draws others in its train; for the slightest cause of movement, a shout or the smallest shock may cause the fall of enormous avalanches." A circumstance of which M. Dufour was a witness confirmed him in his views. A meadow of several acres in extent had been prepared at Morges for skaters by covering it with water, which froze while the heavens were covered. One night the sky cleared off, and M. Dufour noticed a sensible fall in the thermometer. Immediately afterwards he heard crackings in all directions, due to the contraction of the ice from the increased cold, and numerous splits were observable. That phenomenon is precisely analogous to what occurs when the heavens clear up and cause the fall of avalanches.

THERMODYNAMICS.

By HENRY T. EDDY, C. E., Ph. D., University of Cincinnati.

Written for VAN NOSTRAND'S MAGAZINE.

I.

1. INTRODUCTORY.—In 1824 Carnot * laid the foundation of the modern theory of Thermodynamics in the principle that when a heat engine performs work continuously in such a manner that the working substance returns periodically to its initial condition as to temperature, pressure, etc., then during each of these periods or cycles, the working substance must receive heat at a higher and reject it at a lower temperature; and that the work done in the cycle depends in no way upon the working substance, but only upon the temperatures at which the heat is received and rejected.

He further showed that the cycle of maximum efficiency, between two given temperatures, is that in which all the heat received by the working substance is received at the higher temperature, and all the heat rejected, is rejected at the lower temperature. And hence, such a cycle is termed a cycle of Carnot.

Carnot provisionally accepted the hypothesis that heat or caloric is an indestructible material substance, but he evidently regarded this as a doubtful hypothesis.

This error vitiated his reasoning in several regards; in particular, he supposed the amounts of heat received and rejected to be equal, as they must be, if heat is a substance; but the points above enumerated stand to-day as a part of accepted scientific truth.

In 1834, Clapeyron † pointed out the fact, that Watt's diagram of energy is a geometrical expression of the quantities involved in the cycles treated by Carnot. But it was not possible to treat the cycles of Carnot with entire correctness, until the nature of heat itself had been discovered.

As early as 1798, Rumford ‡ had announced, as the result of his experiments in boring cannon, that heat cannot be a substance, and that it was quite impossible for him to conceive it to be anything except motion; but it was not until 1840, or later, that it was proved with entire completeness, by the laborious and elaborate series of experiments of Joule, that heat is energy.

In 1850, Clausius * first made use of Carnot's investigations, as the point of departure for a correct theory of thermodynamics, in accordance with the law of the equivalence of heat and energy.

Clausius † has stated Carnot's principle, thus corrected, substantially as follows:

Whenever a quantity of heat is converted into work, and the working substance is finally found in its initial condition, another quantity of heat must, during the cycle, be transferred from a hotter body or source, to a colder body or refrigerator; and the amount of the heat so transferred, depends only on the temperatures of the two bodies between which the transfer is effected, and not on the nature of the body through which the transfer is effected.

This is Carnot's principle, and is a direct consequence of the second law or principle of thermodynamics, which will be subsequently demonstrated. It is often incorrectly spoken of as being itself the second principle.

In 1849, however, Rankine had derived the general equation of the mechanical action of heat from mechanical considerations based on his "hypothesis of molecular vortices," as to the constitution of matter.

The theory of heat was, during the next few years, very fully worked out and applied by the independent investigations of Rankine, Clausius and Thomson, each of whom has sought to deduce Carnot's cycle (and hence the general equation of thermodynamics) from some physical axiom or hypothesis. We shall discuss these hereafter, and wish merely to state in this connection, that in 1853,

* Reflexions sur la puissance du feu, et sur les machines propres à développer cette puissance, par Sadi Carnot. Paris, 1824.

† Journal de l'Ecole Polytechnique, tome XIV.

‡ Philosophical Transactions, 1798.

* Poggendorff's Annalen, 1850.

† Pogg. Ann., 1854, Vol. XCIII.

Rankine* announced a general law of energy, which seems to afford a better basis for the theory of thermodynamics than any other axiom or hypothesis heretofore proposed.

But being apparently much absorbed in his molecular hypotheses, he has neglected to sufficiently enforce and explain this most valuable conception. As a consequence, the more abstruse and less useful, though more fully explained, axioms of Clausius and of Thomson, furnish the basis of every published treatment of this subject, while if Rankine is referred to at all, only his very words are quoted, showing that their scope is not fully grasped.

We propose in the following treatment to use Rankine's ideas as the basis of reasoning, but to use the analytic forms of such investigators as may appear convenient, among whom may be mentioned, besides Rankine, Clausius and Thomson. the more recent investigators Maxwell, Zeuner, Hirn and Boltzmann.

The reader will find a minute historical statement of the progress of the dynamical theory of heat in Tait's "Sketch of Thermodynamics," and other historical matter in Clausius' *Mémoire, in Poggendorff's Annalen*, Nov., 1863, Upon an Axiom in the Mechanical Theory of Heat.

2. Work, Energy, Heat.—A unit of work is the amount of work performed in raising a unit of weight through a unit of height against gravity.

Energy is capacity for performing work, and is measured in units of work.

An absolute unit of heat is the quantity of heat required to raise a unit of weight of water at its maximum density (39°.1F.) through one degree of temperature.

3. The First Law of Thermodynamics.—Heat is energy, and has capacity for performing work, so that the number of units of work which can be performed by given quantity of heat is proportional to a the number of units of heat in that quantity.

The words "can be performed" in this statement refer to the fact, stated in Carnot's principle, that we find insurmountable difficulties in attempting to convert *all* of a given quantity of heat into work, but the part of any given quantity of heat which we do succeed in converting into work is so converted in as real and exact a sense as that in which, water, for example, is converted into steam.

However, there is no such intrinsic difficulty in converting work into heat, and the results of Joule's final experiments he has stated thus:

1°. The quantity of heat produced by the friction of bodies, whether solid or liquid, is always proportional to the quantity of work expended.

2°. The quantity of heat capable of increasing the temperature of a pound of water (weighed in vacuo and taken at between 55° and 60°F.) by 1°F. requires for its evolution the expenditure of a mechanical force (*i.e.* an amount of work) represented by the fall of 772 lbs. through the space of one foot.

The two experimental truths which underlie all modern physics are the indestructibility of matter and of energy; which may be otherwise stated by saying, that the total mass of the universe is constant, and the total energy of the universe, actual and potential, is also constant.

The experimental verification of the first law of thermodynamics was a principal step in proving the indestructibility of energy.

From this first law we see that heat may be measured in units of work as well as in absolute units.

4. The Second Law of Thermodynamics.—Any effect whatever which is caused by heat, and which is measured in units of work, is proportional to the number of units of heat producing that effect.

The second law is made by Rankine to depend upon the first law in substantially the following manner:

Heat is energy, but energy is a thing of such a nature that its parts are like the whole, and it is unchanged in any particular, except in magnitude, by subdivision or by multiplication; hence, heat also possesses the same characteristics, and any effect caused by heat is proportional only to the amount of heat acting, pro-

*A Manual of the Steam Engine and other Prime Movers, art. 344.

vided the effect is measured in units of work.

This is otherwise stated by saying that the effect, so measured, of any given quantity of heat is the sum of the separate effects of any parts into which that quantity of heat may be supposed to be divided.

The most natural conception of heat, and one which assists the thought, is to regard heat as the energy of molecular or atomic motion.

Various attempts* have been made to derive the fundamental equation of thermodynamics resulting from the second law from the first law, by known mechanical principles.

But Tait says,† respecting these attempts, that they virtually assume, in course of the demonstration, a consequence of the law to be proved, and hence are inconclusive.

Clausius incidentally referred to this matter in his address at the 41st meeting of German Naturalists and Physicists at Frankfort in words to this effect:

"Besides, there is a second principle, which is not yet contained in the first, but requires a special demonstration."

Rankine however states‡ that "Carnot's principle (*i.e.*, the second principle) is not an independent principle in the theory of heat, but is deducible as a consequence from the equations of the mutual conversion of heat and expansive power" (*i.e.*, from the first principle). The demonstration of this which he has given rests upon his hypothesis of molecular vortices.

So far as can be now seen the demonstration of the second law above given, which is dependent only upon the known properties of energy, and which is, simply, one case of a like proposition, enunciated by Rankine, respecting energy in general, is valid and complete, and obviates the necessity of any intricate analytic processes which must of necessity assume the molecular constitution of matter in some general way at least.

It is to be further noted in this connection that universal experience attests the fact that heat, from whatever source, is identical in its general nature and properties, just as energy is, which fact is confirmatory of the above demonstration.

It is to be understood that the necessity for the existence of the second law is to afford a basis for Carnot's principle, previously stated, and that any truth which affords such a basis is called the second law.

5. Other Forms of the Second Law.—It may be useful to consider briefly two other statements of the second law which are proposed as physical axioms and are intended to command our assent from their self evident truth.

1°. "Heat cannot of itself pass out of a colder into a hotter body." (Clausius, 1850). This axiom does not contradict Prevost's Theory of Exchanges, but simply states that in the exchange of heat between two bodies the colder body will receive more than the hotter body. Clausius has, subsequently, restated it thus: "It is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body to another at a higher temperature."

2°. "It is impossible, by means of inanimate material agency, to derive mechanical effect from any portion of matters by cooling it below the temperature of the coldest of the surrounding objects. (Thomson 1851).

All that can be said respecting these proposed axioms is, that they are not self evident, and so are not axioms at all. Indeed, Clausius* has devoted a long memoir to showing that the action of lenses in concentrating rays of heat will not probably vitiate his axiom.

Besides these axioms, various molecular hypotheses have been advanced, from which Carnot's principle has been derived.

1°. Hypothesis of Molecular Vortices. (Rankine, 1849).

2°. Hypothesis of Circulating Streams of any figure whatever. (Rankine, 1851 and 1855) †.

3°. Hypothesis of Periodic Motions. (Boltzmann, 1866.‡)

4°. Hypothesis of Quasi-Periodic Mo-

* The second proposition of the Mechanical Theory of Heat deduced from the First, by C. Szily. Philosophical Magazine, Jan., 1876.
On the Second Law of Thermodynamics in Connection with the Kinetic Theory of Gases, by H. S. Burbury. Phil. Mag. Jan., 1876.
Boltzmann, Sitzungsberichte der Weiner Akad. Vol. LXIII.

† Sketch of Thermodynamics, 2d Edition, Edinburgh, 1877, p. 38.

‡ Phil. Mag. Series IV, Vol. VII.

* Pogg. Ann., January, 1864. Vol. CXXI.

† Phil. Mag. series IV, Vol. XXX.

* Sitzungsberichte der Weiner Akademie, Vol. LIII.

tions. (Clausius, 1871† and more recently).

THE THIRD LAW OF THERMODYNAMICS.

6. The quantity of heat existing in a unit of weight of a given substance depends only on the temperature of the substance, and this quantity of heat is proportional to the absolute temperature of the substance; *i. e.*, the quantity of heat is proportional to the difference between its temperature in its present state, and its temperature when in a state devoid of heat-energy.

This law may be regarded either as an axiom, or as defining the method to be adopted in measuring temperature.

That there is a condition in which any substance is devoid of energy in the form of heat must be admitted, unless we can regard the heat-energy of a unit of any substance whatever as inexhaustible.

All the difficulties which have heretofore attended the demonstration of the second law have equally beset the definition of temperature.

Rankine† would restrict the third law by making it read, "the quantity of heat existing in a unit of weight of a given substance, while it remains in the same condition, solid, liquid or gaseous, depends only on its temperature," etc., etc; but Clausius‡ has shown that Rankine, from his own principles, would be obliged to remove that restriction, and agree that the quantity of sensible heat in a body does not depend upon the state of aggregation, solid, liquid, or gaseous in which it is.

Clausius‡ has also shown, that this third law is a consequence of Carnot's principle, and to this demonstration we may recur after we have discussed that principle.

Indeed, so called axioms have been proposed as the basis of Carnot's principle, which include at once both the second law and the third also; viz:—

1°. "The mechanical work which heat can perform in any change of arrangement of the parts of a body, is proportional to the absolute temperature at which the change takes place." (Clausius.*)

By "change of arrangement of the parts of a body" is meant expansion, fusion or vaporization, or the reverse of either of these, as well as chemical decomposition; and the words "can perform" refer to the case in which such variations of state are effected in a reversible manner, *i.e.* there is no radiation or conduction or the like.

2°. "If the temperature be infinitesimal, the quantity of energy (in the form of heat) converted into work by an isothermal transformation must likewise be infinitesimal." (Belpaire.†)

Thompson,‡ however, evades this difficulty by proposing a definition of temperature dependent upon Carnot's principle, and it appears that this highly philosophic basis for the scale of temperatures would hardly differ perceptibly from that derived from the third law; but the practical graduation of an instrument on the basis of Carnot's principle has not been yet accomplished by reason of the physical difficulties encountered in the process.

The foregoing remarks convey no adequate idea of the extent, originality and importance of the labors of Thomson and of Clausius, but as no physicist is ignorant of their great discoveries in this and kindred fields, it is not necessary to deal further with the subject in this connection.

7. TRANSFORMATIONS OF HEAT.—When heat is imparted to a body, there are in general three ways in which the energy of the heat is expended.

1°. Temperature is augmented, and the part of the heat so expended still exists in the substance as sensible heat.

This is explained on the molecular theory of matter as actual energy of motion of molecules.

2°. Internal work is performed during the expansion or contraction accompanying the change of temperature.

On the molecular theory this is regarded aswork performed against atomic or molecular forces in causing fusion, vaporization, dissociation or the like, or in preparing the body for such changes.

Work so expended is no longer heat,

* Pogg. Ann. Vol. CXLII.
† Steam Engine, p. 307, 7th Ed.
‡ Pogg. Ann., November, 1863, Vol. CXX.

* Pogg. Ann., 1862, Vol. CXVI.
† Bulletin de Belg., 1872.
‡ Phil. Mag., 1848.

and does not affect the thermometer, and hence is called potential or latent heat. Although latent heat, not being heat at all, is a misnomer, it is nevertheless convenient to retain this term which expresses a part of the exploded hypothesis of the materiality of caloric.

3° External work is performed by the heat during expansion or contraction, under the action of external forces such as pressure. The heat which is thus converted into work is stored up as potential energy of the external bodies which exert pressure upon the substance to which heat is imparted.

The first two forms of energy together constitute the increment of the internal energy, actual and potential, in contrast with the last which is the external work.

The last two forms of energy together constitute the increment of the so-called latent heat or potential energy, in contrast with the first which is the increment of the actual (sensible) heat or actual energy.

In order to express these statements as equations, let the following quantities be expressed in units of work, as they evidently can be by the first law.

Let dh=the total increment of heat imparted to a unit of a given substance.

" ds=the part of dh which increases its sensible heat or temperature.

" dm=the part of dh expended in doing internal work against molecular forces.

" dw=the part of dh expended in doing external work.

" di=the increment of internal energy.

" dl=the increment of latent heat.

Then,
$$di=ds+dm \quad . \quad . \quad . \quad . \quad (1)$$
$$dl=dm+dw \quad . \quad . \quad . \quad . \quad (2)$$
$$dh=ds+dm+dw \quad . \quad . \quad . \quad (3)$$
$$dh=di+dw \quad . \quad . \quad . \quad . \quad (4)$$
$$dh=ds+dl \quad . \quad . \quad . \quad . \quad (5)$$

These equations evidently refer to a unit of *any* substance, homogeneous or otherwise, or to any mixture whether undergoing fusion, solidification, vaporization, condensation, dissociation, or any change, chemical or otherwise.

8. SPECIFIC HEAT.—The total quantity of heat which must be imparted to a unit of weight of a given substance in order to augment its temperature by 1°, or the total quantity rejected in decreasing its temperature by 1° is its specific heat. This is usually expressed in absolute units, but for convenience we shall suppose it expressed in units of work according to the first law.

Calorimetric determinations of this quantity are readily made for any given substance under given conditions of pressure and temperature. If equation (3) be considered for the moment to apply to the case of a unit of a given substance which is increased 1° in temperature, then the only one of the four quantities in that equation which is fixed by that statement is ds, for the others will depend upon the change of volume and the pressure, but by the third law ds is invariable and independent of any consideration except temperature and the kind of substance. Hence this part of the specific heat is called the real specific heat, or the true calorific capacity of the substance.

Let s=the total sensible heat in a unit of weight of a given substance.

" k=its real specific heat.

" t=its absolute temperature.

As before stated, absolute temperature is counted from such an origin as zero, that its temperature vanishes with its sensible heat.

By the third law, $$\frac{s}{t}=k \quad . \quad . \quad . \quad (6)$$

$$\therefore \frac{ds}{dt}=k, \quad \therefore \frac{ds}{s}=\frac{dt}{t} \quad . \quad . \quad . \quad . \quad (7)$$

Equations (7) may also be written in the following forms:

$$\frac{s_2-s_1}{t_2-t_1}=k, \quad \frac{s_2-s_1}{s_1}=\frac{t_2-t_1}{t_1} \quad . \quad . \quad (8)$$

in which the subscripts 1 and 2 refer to any two states of the substance. Equations (7) and (8), which do not contain k, the specific heat of the given substance, are general and refer to any substance whatever.

It is now evident that the third law may also be stated in this form:

Equal increments of sensible heat imparted to any given substance cause equal increments of temperature.

9. THERMOMETERS.—It is well known that the mercurial and other solid and fluid thermometers are inaccurate, because the expansions by which they measure variations of temperature are not uniform; at least, they are inaccurate on the hypothesis that equal expansions are caused by equal variations of temperature, for, on that hypothesis, thermometers made of different substances give discordant results. But with air thermometers, the case is different. Air is very nearly a perfect gas. The definition of a perfect gas is one that fulfills the Law of Gay Lussac, a law expressed by the equation

$$\frac{pv}{t} = \frac{p_0 v_0}{t_0} \quad \ldots \quad (9)$$

in which p, v, t, are respectively the pressure, volume and temperature of the gas at any state, and p_0, v_0, t_0 at same assumed initial state.

In an air thermometer we read, by a scale, the expansion of a confined mass of air, which expansion is caused by change of temperature, and is effected against a constant external pressure. It is easily seen that under all circumstances, we have the general equation

$$dw = pdv \quad \ldots \quad (10)$$

in which dv = the expansion or increment of volume.

$$\text{By (9)}, \quad dw = pdv = \frac{p_0 v_0}{t_0} dt \quad \ldots \quad (11)$$

$$\therefore \frac{dv}{dt} = \frac{p_0 v_0}{p t_0} = \text{a constant} \ldots (12)$$

Hence, in this thermometer, since p is constant, equal increments of temperature, as defined by the third law, cause equal expansions.

10. ISOTHERMAL AND ADIABATIC LINES.—Let Fig. 1 be a diagram in which the different states of a unit of weight of a given substance are represented graphically by its volume v and pressure p, just as is done by the Indicator Diagram of the steam engine. This is a matter so commonly understood that it is not necessary here to explain it in detail. Let $t_1 t_1$ be the isothermal of the given substance at the temperature t_1, *i.e.* $t_1 t_1$ is a line such that when a point moves along it, the co-ordinates, v and p of the point, correctly represent the magnitudes of volume and pressure of the substance, in case the temperature t_1 is unchanged.

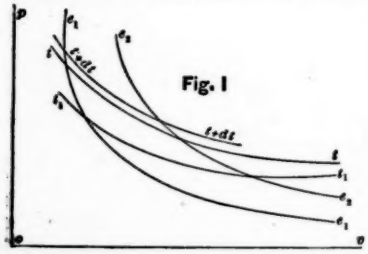


Fig. 1

Similarly let tt be any other isothermal. Again, let $e_1 e_1$ be an adiabatic, *i.e.*, a line representing in the same manner the v and p of successive states of the substance, on condition (not that temperature is constant but) that no heat is imparted to the substance or taken from it. Similarly let $e_2 e_2$ be any other adiabatic.

The forms of the isothermals and the adiabatics for any given substance can, in general, be found from experimental data only, but we shall show subsequently how to make the equations of these curves depend upon a very small number of such data.

If a substance expand along $e_1 e_1$, then

$$\text{by (5)} \quad dh = 0, \quad \therefore dl = -ds = dm + dw \quad . \quad (13)$$

Hence during such expansion sensible heat is withdrawn from the substance and expended in doing work external and internal. Now suppose successive isothermals to be drawn at a distance of 1° from each other, it then appears from (8) that the same quantity of heat disappears, or is withdrawn from the total sensible heat of the substance every time it passes from one isothermal to the next lower isothermal, however that passage be effected, whether along an adiabatic or not.

If a substance is made to pass along an adiabatic no heat is imparted to or withdrawn from the substance by surrounding bodies, but if the passage occurs along any other line, then heat is also imparted to or rejected from the working substance in the form of heat. Suppose, as a most important example, that the substance expands along an isothermal, then heat must be supplied to it in order to preserve the temperature t unchanged. Now since t is constant, in the variation supposed, the total sensible heat is constant also, by the third

law, as expressed in (8). Hence in this case

$$ds=0,\ \therefore\ dh=dl=dm+dw \quad . \quad . \quad (14)$$

Hence it appears that the heat imparted along an isothermal is all expended in performing work, external and internal.

But h, the total heat imparted to the substance during its expansion along t from the adiabatic e_1 to e_2, is measured in units of work by the area included between the curves $e_1\ t\ e_2$, as appears from the fact that if the substance, by help of the heat imparted, be carried along t from e_1 to e_2 and then left to expand along e_2 until devoid of heat-energy, it will, during these two operations, perform an amount of external work greater by the area included between $e_1\ t\ e_2$ than if left to expand along e_1 from its initial state at the intersection of t_1 and e_1 until devoid of energy.

In general, the amount of heat imparted to a substance in order to carry it along any route x from the state 1 to the state 2 is measured in units of work by the area included between x and the two adiabatics e_1 and e_2 infinitely extended, which pass through 1 and 2 respectively.

Now suppose two isothermals drawn very near together, the first at a temperature t, the second at a temperature $t+dt$, then the amount of heat h required to carry the substance along t from the adiabatic e_1 to another adiabatic e_2 at some finite distance from e_1, is not so great as the amount of heat $h+dh$ required to carry it along $t+dt$ from e_1 to e_2 by an amount represented in units of work by the area of the quadrilateral included between the lines t, e_1, $t+dt$ and e_2.

The cause requiring a larger supply of heat along $t+dt$ than along t can only be found in the higher temperature (*i. e.*, the greater amount of sensible heat) existing in the substance along $t+dt$. Hence this is a case to which the second law applies

$$\therefore\ h+dh : h :: s+ds : s \quad . \quad . \quad . \quad (15)$$

$$\therefore\ \frac{dh}{h}=\frac{ds}{s},\ \therefore \text{ by (7)},\ \frac{dh}{h}=\frac{dt}{t} \quad . \quad . \quad (16)$$

$$\therefore\ \frac{h_2-h_1}{h_1}=\frac{t_2-t_1}{t_1},\ \text{or}\ \frac{h_2}{h_1}=\frac{t_2}{t_1} \quad . \quad . \quad (17)$$

From these equations it appears that the amount of heat required in order to carry a substance along an isothermal from one adiabatic to another depends upon which isothermal it is, and the amount required is proportional to the absolute temperature.

11. Definition of Entropy.—During any small variation in the state of a unit of a given substance, *i.e.*, during its passage along any short distance of any route whatever:

Let t = the mean absolute temperature.
" dh = the heat imparted.
" de = the entropy imparted.

Then $$dh=tde \quad . \quad . \quad . \quad . \quad . \quad (18)$$

is the equation which defines entropy. Entropy, which may be roughly defined as "heat divided by temperature," plays a very important role in thermodynamics, which fully justifies its introduction as one of the variables to be used in expressing the state of a substance. Entropy has been used by Tait, Thomson, Maxwell and others, with an entirely different signification, but the term is here used in the sense originally given it by its inventor Clausius*. It is identical in meaning with Rankine's "thermodynamic function" and Zeuner's "thermic weight."

12.—The Law of the Variation of Entropy.—The amount of entropy which must be imparted to a unit of weight of a given substance in order to make it pass from one given state to another (of volume and pressure) is independent of the route by which the passage is effected, and depends only on the initial and final states of the substance.

For let two points be selected on a diagram of volumes and pressures which shall represent the two given states, and let them be designated by 1 and 2. Also let there be two routes from 1 to 2 which are represented by any two curves x and y. Moreover let the curves x and y be cut by a series of isothermals and of adiabatics drawn at any equal or unequal small distances from each other. Then the variations of state which occur along the route x can be represented as exactly as we please, by a series of infinitesimal

* Pogg. Ann. 1865, Vol. CXXV.

variations in a zigzag manner along these isothermals and adiabatics, and the same is true of the variations of state along y. Let us call that zigzag route x', which nearly coincides with x, and similarly that one y' which nearly coincides with y.

Now during any variations along adiabatics either on x' or y', we have

$$dh=0, \qquad \therefore de=\frac{dh}{t}=0, \quad . \quad . \quad . \quad (19)$$

hence, during such variations, entropy is constant. But during any variation along an isothermal in the route x' between a pair of successive adiabatics, we know by (17) that dh_x, the heat imparted is proportional to t_x the absolute temperature, and during the variation along y' between the same pair of adiabatics dh_y, the heat imparted, has the same ratio to t_y its absolute temperature.

$$\therefore de=\frac{dh_x}{t_x}=\frac{dh_y}{t_y},$$

$$\therefore e_2-e_1=\int_1^2\frac{dh_x}{t_x}=\int_1^2\frac{dh_y}{t_y}$$

$$\therefore e_2-e_1=\int_1^2\frac{dh}{t} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

which expression depends only on the initial and final states of the substance.

EQUI-DISTANT ADIABATICS AND ISOTHERMALS.

13. By art. 10, $\dfrac{h_2-h_1}{h_1}=\dfrac{t_2-t_1}{t_1}$ . . (17)

Let the difference of temperature (or the distance) between successive isothermals be one degree

$$\therefore t_2-t_1=1 \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Also, let the excess of heat required in order to make the substance pass along t_2 from e_1 to e_2 above that required in order to make it pass along t_1 from e_1 to e_2 be one unit of work,

$$\therefore h_2-h_1=1 \quad . \quad . \quad . \quad . \quad (22)$$

$\therefore$ by (17), $h_1=t_1$, $h_2=t_2$, and $h=t$ . . (23)

Hence, the successive members of a system of adiabatics may be fixed in such a way by (21) and (22), and at such a distance apart, that the number of units of heat which must be imparted to the working substance in order to make it pass along a given isothermal from any adiabatic to the next, is numerically the same as the absolute temperature.

From (22) it appears that the quadrilateral area included between any pair of successive adiabatics and any pair of successive isothermals, is one unit of work.

By (23), $h=t$, $\therefore h\div t=1$ . . . . (24)

Hence, the entropy imparted in passing along any isothermal whatever, from one adiabatic of this system to the next, is unity. But the passage need not be restricted to an isothermal, for along an adiabatic

$$dh=0 \quad \therefore \frac{dh}{t}=0 \quad . \quad . \quad . \quad . \quad (19)$$

Hence, entropy does not vary along an adiabatic, hence the adiabatics are isentropics.

The successive isothermals should be numbered so that temperature vanishes when the substance is without sensible heat. And the successive adiabatics should be so numbered that entropy vanishes when the substance has no energy actual or potential.

14. INTEGRATING FACTOR.—One of the factors which will render integrable every differential expression for the heat imparted, such as (3), (4) or (5), is t^{-1}. For suppose, as in art. 12, that there are two routes x or x' and y or y' leading from 1 to 2; then, as shown in art. 10, the heat which must be imparted to make the substance pass from 1 to 2 along x, is the area included between the curves e_1 x e_2, and the heat imparted along y is the area between the curves e_1 y e_2. These areas are not in general equal, consequently $h_2-h_1=\int_1^2 dh$ is not in general integrable until the route from 1 to 2 is given, for its value is dependent upon that route. But we have shown that $e_2-e_1=\int_1^2 t^{-1}\,dh$ . . (20) is not dependent upon the route from 1 to 2.

But any differential expression which is a function of several variables and their differential coefficients, and which, when integrated between assigned limits, can have but a single value, and this value dependent alone upon those limits, is therefore a differential expression which can be integrated (*i.e.*, summed) between those limits. Hence, such an expression fulfills all the requirements of integrability. But these requirements

are expressed algebraically by the so called "equation of condition of integrability," which holds in case of a total or exact differential. Hence, (20) is an integrable expression, and $t^{-1}\,dh$ is an exact differential which fulfills all the conditions of integrability, algebraic, or otherwise which can exist.

It is possible by the help of the second law to show algebraically, that t^{-1} is an integrating factor, and this is done subsequently in certain cases; but it appears more in accordance with physical reasoning, to show why t^{-1} is necessarily an integrating factor, than to divert the attention to the forms employed in the algebraic process for proving the same thing. The foregoing proof is that usually given to show that the forces which act towards the fixed centers (as do the forces of nature) have always a definite potential, or force function, for every point of space, and that the work done against such forces in moving a particle from 1 to 2, is not dependent upon the path. This matter is treated in detail by Clausius.*

* Dingler's Polytechnic Journal, Vol. CL.

TURBINE WHEELS.

ON THE INAPPLICABILITY OF THE THEORETICAL INVESTIGATIONS OF THE TURBINE WHEEL, AS GIVEN BY RANKINE, WEISBACH, BRESSE AND OTHERS, TO THE MODERN CONSTRUCTIONS INTRODUCED BY BOYDEN AND FRANCIS.

By Prof. W. P. TROWBRIDGE, Columbia College.

Written for Van Nostrand's Magazine.

The increasing importance of the Turbine wheel as a motor, gives especial interest to everything relating to the history of its development; and although the improvements which have been introduced from time to time have been brought about by practical men in the first instance, yet the professional engineer is often called upon to investigate the merits of special designs, or to decide upon the theoretical performance of particular wheels.

To enable him to do so, it is essential for this as for all other motors, that the scientific principles of construction should be determined and introduced into text books of instruction for general information. We accordingly find full, and often tediously minute, mathematical investigations following the introduction of the Reaction Turbine of Fourneyron, in France years ago.

The improvements in construction and design made by our own countrymen, Uriah A. Boyden and James B. Francis, marked a second period in the useful applications of the Turbine, no less important and radical in some respects than those which characterized the invention of Fourneyron. For the services thus rendered to industry, the names of Boyden and Francis deserve more universal recognition than they have yet received. My object is not, however, to assert the claims to which they are entitled, but to show, that the mathematical investigations which have been alluded to as following the introduction of the Fourneyron wheel and which are still given in text-books as the accepted expositions of the subject of Turbines, are insufficient and inapplicable to the modern constructions which are founded upon the principles introduced by Boyden and Francis.

In other words, while the Turbine wheel has been improved and its efficiency greatly increased, we look in vain for suggestions which might lead to such improvements, in the treatises on Turbines, by Rankine, Weisbach, Bresse and others, which have been and are still most frequently studied for information on the subject.

Mr. James B. Francis remarks in his well known work entitled "Lowell Hydraulic Experiments" that "the turbine has been an object of deep interest to many learned mathematicians, but up to this time the results of their investigations, so far as they have been published,

have afforded but little aid to Hydraulic Engineers."

How far this statement may be literally true or not, it is unnecessary to discuss: but interpreted as the conclusion of Mr. Francis, in regard to the improvements which it was his object to describe in the Lowell Hydraulic Experiments, the statement seems undeniable; because it may be shown, I think, that if Boyden and Francis had followed strictly the rules of construction, laid down in the works which have been alluded to, they would have failed in their efforts to construct Turbines giving any considerable increase of efficiency over the old Fourneyron and Fontaine or Jonval wheels of European design and construction.

Writers on Turbines had, up to that time, insisted upon a mechanical axiom or principle of construction which, although exemplified in the Fourneyron Reaction wheel, was directly violated by Boyden and Francis in their constructions.

Their success was a sufficient demonstration of the correctness of their practice. Thus while their improvements have been universally accepted and followed, at least in this country, the student and the engineer are continually confronted with the old mathematical theories; and it is not surprising that others besides Francis have been puzzled in their efforts to apply old formulas to the newer constructions.

The theorem or axiom insisted upon by these writers was that the "water must enter the wheel *without shock*," and hence, the mathematical condition that the *tangential velocity of the wheel, where it receives the water, and the corresponding component of the velocity of the entering water* must be equal; the effect of which is to prevent all *impulsive* effects of the entering water.

Taking the old Fourneyron wheel as an example, the buckets or floats were given a radial direction at the point where they received the water, and the relative velocities were made such by the mechanical conditions assumed, that the water entered the wheel *radially*, with a tangential component equal to the velocity of the bucket at this point. Its effects in producing *mechanical work* were thus made to depend solely upon the subsequent deviation which it experienced in passing through the wheel. This device was to avoid the *shock* of the entering water.

It is to be noted that both Rankine and Weisbach, in discussing the impulse and reaction of jets of water upon moving *vanes*, make no reservation in regard to the *shock* due to impulse, but demonstrate that water may impinge at any angle and with any relative velocity upon vanes and, by a suitable arrangement of curvature and velocities, may have all the energy destroyed; and a perfect efficiency may be obtained.

It is difficult to understand why in their discussions of the *Turbine* wheel, they insist on a different principle, and lay down a mechanical axiom at variance with these demonstrations.

The following discussion is intended to apply to the constructions introduced by Boyden and Francis, but, as will be seen in the development, the general formulas deduced are applicable to nearly all water-wheels, by making simple and proper suppositions in regard to the quantities which enter into them.

The Turbine wheel consists essentially of two rims or crowns firmly attached to an axis, numerous curved vanes or *buckets* being fixed between the crowns, and the main stream of water which passes through the wheel is divided by guide blades, not attached to the axis, into numerous jets or smaller streams, which impinge upon these buckets simultaneously at all points of the circumference, and produce motion in the wheel.

It is sufficient, in discussing the action of the water on the wheel, to take one of these jets separately with its guide blade and bucket, inasmuch as all the jets act in the same manner.

The problem then, is to analyze the action of a single jet upon a curved bucket, and to find the mechanical work performed and the conditions of maximum efficiency.

To do this, it is proper to enumerate the axioms or theorems of the mechanics of fluids, which, are applicable to the subject, and which require no special proof. The proof of these theorems being derived from general observation and experience.

These theorems are as follows:

1*st*. When a surface moves in a given

direction under given pressures, the component pressures in all directions, except that of the motion, are neutralized, either by reciprocal actions, or by the fixed surfaces which guide the moving surface.

Therefore, in considering mechanical work done by given pressures acting upon moving surfaces, it is necessary to take into account those components only of the pressures which act in the direction of the motion, friction being neglected.

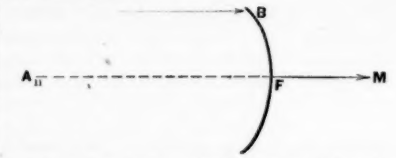
2nd. In whatever direction a surface be moving with reference to the earth, if a fluid moves along this surface in a direction opposite to the motion of the surface, and with a relative velocity equal to the velocity of the surface with reference to the earth, the fluid will be at rest with reference to the earth.

3rd. A fluid stream on striking a smooth surface, at any angle whatever, is not reflected like a solid, but flows along the surface. If the surface be fixed, and the stream be confined in a channel of uniform dimensions before and after striking the surface, the velocity of the stream will remain unaltered, friction not being considered.

If the surface be moving, the velocity under the same conditions after striking the surface will be the *relative* velocity of the surface and fluid before impact. If, for instance, a fluid jet impinge perpendicularly upon a plane surface moving with any velocity in the same direction, the *relative* velocity will be the difference of the two velocities and this will be the velocity with which the stream will flow along the surface.

This initial difference of velocities gives rise to a pressure due to *impulse*, the direction of the pressure being always normal to the surface at the point of impact.

4th. If the surface be curved, as in the following sketch, the same phenomena occur except that the relative velocity of the particle of the stream is not entirely destroyed until it reaches a point at which it is moving at right angles to the direction of the motion of the surface. For example, if the stream AB impinge upon the curved surface represented in the sketch, the surface moving in the direction FM, the difference of velocities of the surface and any particle will be wholly destroyed when the particle reaches the point F, the whole effect up to this point is one of *impulse* and is the same as if the stream impinged directly upon the surface at F and was deviated instantaneously at right angles to the motion FM.



5th. After the particle passes the point F it flows along the curved surface without any further change of relative velocity; a change of curvature, having no effect to change the velocity of *flow relatively to the surface.*

A change of curvature, however, causes a change of *direction* of the motion of the fluid with reference to the motion of the surface and the reaction of the fluid or resistance which it offers to this change, produces a pressure, the pressure due to *reaction.*

If a fluid *vein*, having a fixed direction and velocity with reference to the earth, impinge upon a surface which also has a motion with reference to the earth, the energy imparted to the surface may thus be separated into two parts: that due to the *impulse* and that due to *reaction.* The effect of impulse may be represented by the expression $\overline{M.x.u.}$, in which M is the mass of fluid striking the surface in one second, x the relative velocity in the direction of the motion of the surface, *i.e.*, the difference between the velocity of the surface and the component of the velocity of the fluid in the same direction.

The second effect, due to reaction, results from the deviations which the particles of the fluid vein undergo while in contact with the surface, after they have attained the velocity of the surface. This effect is measured by an expression precisely similar to the first, viz., Mxu, in which M is the mass, as before, x, the relative velocity *imparted* to the fluid in a direction opposite to the motion, and u the velocity of the surface. The total effect being

$$W = M(xu + X_1 U)$$

This being the energy in foot-pounds imparted to the surface.

If all the energy of the vein is destroyed so that it comes to rest with reference to the earth, it is evident that x_1 must then be equal in amount, and directly opposite to u, and the efficiency of such an arrangement will be unity; that is, all the energy of the fluid stream will be transferred to the surface, and we shall have

$$W = M\,(xu + X_1 U) = \frac{M v_1^2}{2}$$

$$E = 1$$

v_1 being the velocity of the fluid with reference to the earth before striking the surface.

If there is no effect from impulse, the relative velocity of the particles of the fluid vein and the surface in the direction of the motion of the surface must be zero—or $x = 0$.

The whole effect produced must then be from reaction, and it is evident that for an absolute maximum of effect, $M X_1 U$ must be equal to

$$\frac{M v_1^2}{2} \text{ or } X_1 U = \frac{v_1^2}{2}.$$

If the surface be a plane surface, so that there is no deviation of the vein after it strikes the surface, the whole effect will be due to impulse, and $W = M \,.\, x \,.\, u$, will be the total work performed, or the total energy imparted to the surface.

These three general cases,

$W = M(ux + X_1 U)$. impulse and reaction,

$W = M X_1 U$ . . . reaction,

$W = Mxu$ . . . impulse,

explain the whole theory of the action of a stream of water, or of multiple streams in producing motion in water-wheels; not including, however, those cases in which water acts by its weight. In these cases, the first expression may be applied by substituting for the expression representing reaction, $\overline{M X_1 U}$, a form of expression representing the action of weight falling through a given distance, and the expression

$$W = M(xu + Wh)$$

will be the general expression for the energy of Overshot and Breast wheels, W being the weight of water entering the wheel in one second, $\overline{M x u}$ being the work of impulse of the water entering the buckets and h the height of fall.

Nearly all modern turbine wheels are constructed after one of three types, or of some combination of these types. They are illustrated in the following sketches, and usually receive the designations of

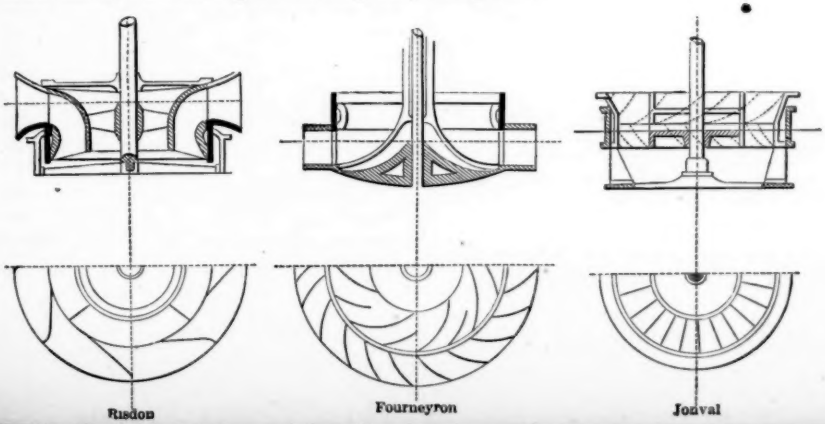
Outward flow wheels,
Inward flow wheels, or center vent wheels.
Parallel flow wheels.

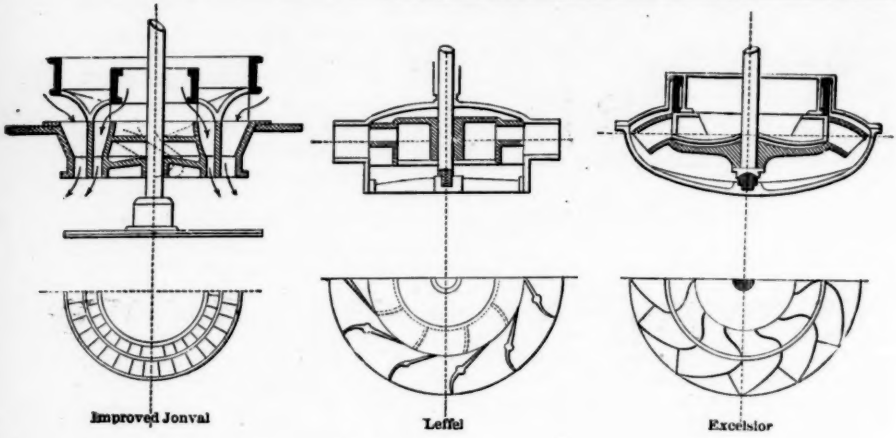
In the outward flow wheel of which the Fourneyron reaction wheel is the earliest type, the water flows usually downward through a tube or conduit, and is diverted by fixed guide blades in an outward direction, or from the axis of motion; the form of the fixed guides giving the water a tangential whirl as it enters the wheel.

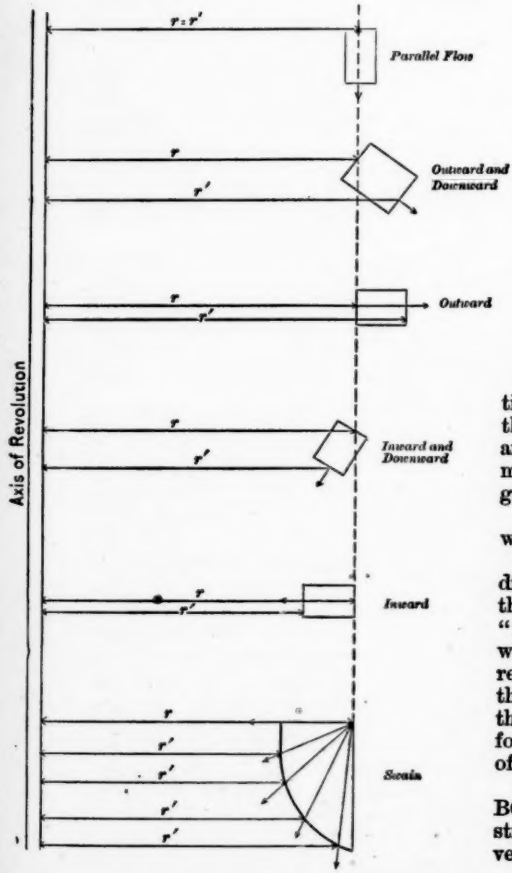
In the inward flow, the water flows first in the direction of the axis, usually downward, and is then diverted by fixed guide blades inwardly, or toward the axis of motion, the same fixed guides giving the water, as before, a tangential whirl as it enters the wheel.

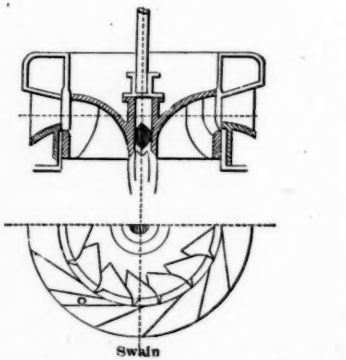
In the parallel flow wheel, the water moves parallel to the axis of motion, before and after it passes through the wheel; the fixed guide blades, as before, giving the water a tangential whirl.

The principle types of some of the best wheels in use are represented in the following cuts:









The position of the buckets, the direction of the water entering the wheels and the relative values of r and r', the outer and inner radii of the wheels, may be seen more clearly by reference to the diagram given opposite.

To investigate the problems connected with these wheels more minutely:—

Suppose a jet or stream, having a given direction and velocity with reference to the earth, to strike a surface, a vane or "float," all points of which have a motion with a given direction and velocity with reference to the earth; the component of the velocity of the jet in the direction of the motion of the surface may be easily found by the theorem of the parallelogram of velocities.

Let ABC (Fig. 1) be the direction, and BC$=v$, the velocity of a jet of water striking the surface BE, which has a velocity BD$=u$.

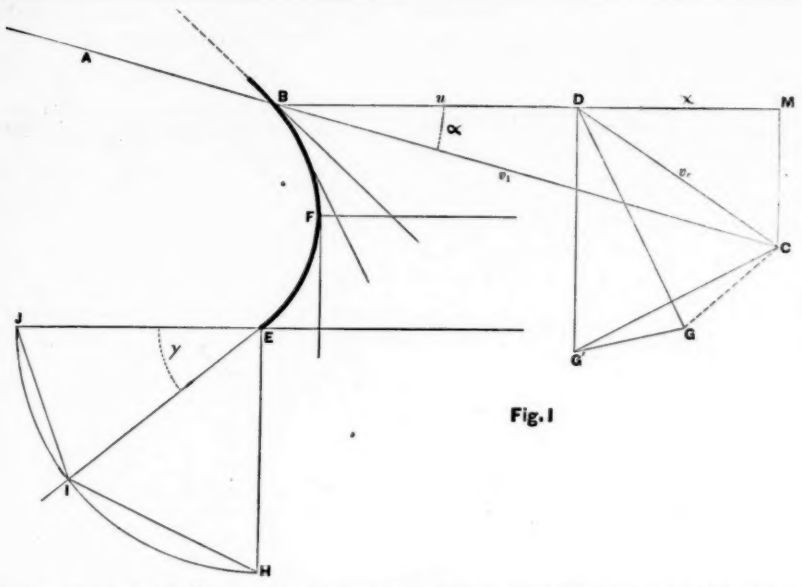


Fig. I

Let the angle DBC=a be the angle made by the direction of the jet with the direction of the motion of the surface.

Then $v_1 \cos. a$ will be the component of the velocity of the stream in the direction BD of the motion of the surface; DM=x will be the difference of velocity in the direction of the motion of the jet and surface and DC the total relative velocity of the jet and surface, in magnitude and direction.

If we draw a tangent to the surface at B and draw the line DG, parallel to this tangent, making DG equal to DC, CG will be the instantaneous deviation due to the impact.

When the jet reaches the point F of the surface, a point at which its direction is perpendicular to the motion of the surface, if we draw the line DG′ parallel to the tangent at F or perpendicular to the motion of the surface, the total deviation of the jet will be CG′ and its projection on BDM, or its component in the direction of the motion BD, will be

$$X=(v_1 \cos. a-u)$$

The relative velocity x_1 of the jet *in the direction of the motion* is not entirely destroyed until it reaches the point F at which its direction is perpendicular to the direction of the surface BD, and hence we may call the total effect up to this point the impulse of the fluid.

If M be the mass of fluid which passes in a unit of time, the momentum of the fluid with reference to the direction of motion of the surface will be

$$Mx=M(v_1 \cos. a-u)$$ and

the mechanical work performed by this momentum will be

$$Mxu$$

After the jet passes the point F, the only pressure that it can exert on the surface will be due to a change of direction or deviation, the *reaction* being the pressure exerted.

If the fluid passes off the surface at E, in the direction of the tangent, at E the amount of the additional deviation may be found by drawing EI in the direction of the tangent, and EH in the direction of the jet at F, making EH=EI=DC.

Then HI will be direction, and will represent the pressure of the jet due to *reaction*, and the projection of this line on the direction of motion, JE, will be the component of this pressure in the direction of the motion.

Representing by γ the angle of departure, made by the tangent IE with the direction of motion, this component will be

$$\overline{IE} \cos. \gamma$$

or calling IE=DC the *relative* velocity, V_r this component will be

$$V_r \cos. \gamma,$$

and the mechanical work performed per second will be the component pressure $M\overline{V_r \cos. \gamma}$ multiplied by U, the velocity of the surface, or

$$M\overline{V_r \cos. \gamma}\, U.$$

The total work performed by the jet upon the surface due to *impulse* and *reaction*, will be

$$W=M(XU+\overline{V_r \cos. \gamma}\, u)$$

the sum of the two effects.

If $\gamma=o$, the work of reaction will be $M V_r U$, and the total work

$$W=M(XU+V_r U).$$

Since V_r is a function of u, we may find under what circumstances this work will be a maximum.

The problem resolves itself into this: in the known triangles BCD, and BCM required to find the position of the line DC such that $W=u(x+v_r)$ shall be a maximum.

By applying the methods of Calculus it will be found that the expression will be a maximum when $u=v_r$

That is for a maximum,

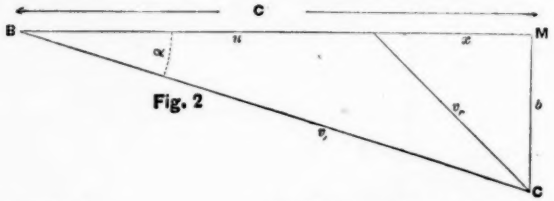


Fig. 2

Let BM=C CM=b

then

$$x=\frac{c^2-b^2}{2c}\quad v_r=u=\frac{c^2+b^2}{2c}$$

and

$$xu+v_r u=xu+u^2=\frac{v_1^2}{2}$$

The same may be demonstrated by geometry. When $\gamma=o$ and $v_r=u$ the fluid in leaving the vane or surface has a motion in a direction opposite to that of the vane and with a relative velocity equal to that of the vane; hence it will be at rest with reference to the *earth:* and all its energy will have been imparted to the vane.

Since v_1 is the original velocity of the fluid with reference to the earth, its energy is $M\frac{v_1^2}{2}$; but we have under the conditions of the maximum just found

$$(W)_{\text{max.}}=M(xu+u^2)=M\frac{v_1^2}{2}$$

or the maximum work is the whole energy of the fluid, and the efficiency

$$E=\frac{\frac{Mv_1^2}{2}}{\frac{Mv_1^2}{2}}=1$$

If the angle γ is not zero, then the work performed cannot be an absolute maximum, or the efficiency cannot be unity, because the fluid will not leave the vane with a direction and velocity equal and opposite to the direction and velocity of the vane. In other words, the fluid will, after leaving the vane have a motion with reference to the earth, the energy due to which will be lost.

The value of this lost work is easily found. In practice the angle γ is always very small.

If a parallelogram (Fig. 3) be drawn, having for sides, the relative velocity EI=v_r and ET=u the diagonal ER for very small angles will be practically $v_r \sin. \gamma$ which will be the velocity of the stream after it leaves the vane at E. The energy due to this velocity will be $\frac{Mv_r^2 \sin.^2\gamma}{2}$ and the general value of the work will be

$$W=M\left(xu+u^2-V^2_r\,\frac{\sin.^2\gamma}{2}\right)$$

$$W=M\left(\frac{v_1^2}{2}-\frac{v^2_r \sin.^2\gamma}{2}\right)$$

and the efficiency

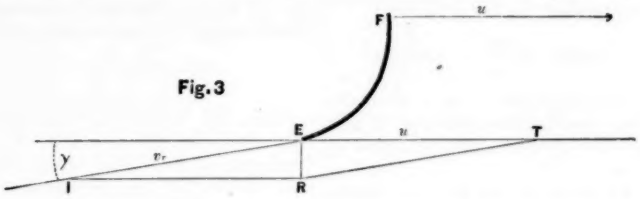


Fig. 3

$$E=\frac{M\left(\frac{v_1^2}{2}-\frac{v_r^2 sin.^2\gamma}{2}\right)}{\frac{Mv_1^2}{2}}$$

or

$$E=1-\frac{v_r^2}{v_1^2}sin.^2\gamma$$

Assuming now that the velocity u was that which gave the absolute maximum when $\gamma=o$ or $u=v_r=\frac{v_1}{2\cos.\ a}$ (see Fig. 2) we have by substituting this value of v_r in the expression for the efficiency

$$E=1-\tfrac{1}{4}\frac{\sin.^2\gamma}{\cos.^2 a}.$$

From this it is apparent that the efficiency is increased by making a and γ small.

If $a=o$ then the stream impinges at right angles to the vane or in the direction of its motion, in which case $x=v_1-u$ and $u=\frac{v_1}{2}$ for the maximum efficiency, and the expressions for the efficiency become for $\gamma=o$

$$E=I$$

and for γ a small angle

$$E=I-\tfrac{1}{4}\sin.^2\gamma$$

Variations from maximum work for variations of u.

If we describe a circle (Fig. 4) on $BM=v_1\cos.\ a$ and lay off x and u in such a way that $OC=v_r=u$, we see that the

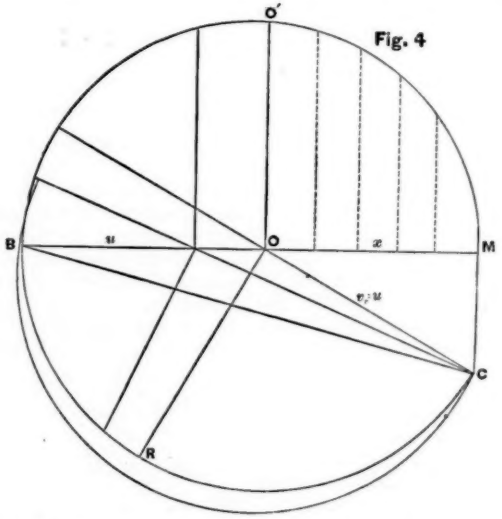


Fig. 4

two terms of the expression $xu+v_r u$ will vary as the squares of the ordinates (of the circles) OO′ and OR respectively, and the positions of these ordinates are such that the variations of their squares, *i. e.*, $\overline{OO'}^2=xu$ and $\overline{OR}^2=v_r u$ will be a minimum, or in other words, small variations of u from the value giving the absolute maximum, may occur without affecting greatly, the value of the work.

Variations from maximum efficiency for variations of the angles a and γ.

In the expression for the efficiency

$$E=1-\tfrac{1}{4}\frac{\sin.^2\gamma}{\cos.^2 a}$$

found on the supposition that γ is small. If a is also small, the efficiency is not greatly affected by very small changes in these angles.

a being a fixed angle by the conditions of the problem; if γ be variable within small limits, the variations will not affect greatly, the work performed.

Method of finding the work performed and the conditions of absolute maximum when the vane or surface revolves around a fixed axis.

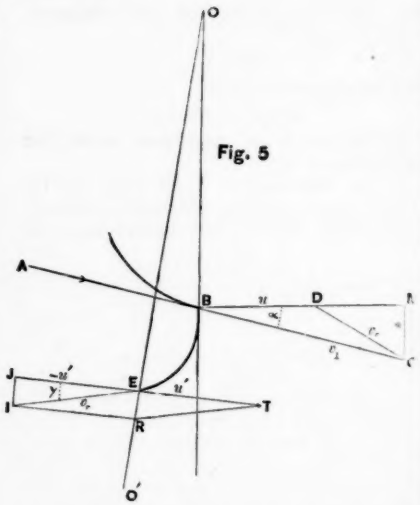


Fig. 5

Let ABC (Fig. 5) be the direction of the jet with reference to the earth; BD $=u$ the velocity of the point B at which the tangent to the vane coincides with the radius OB of the circle, around whose center the motion takes place: EJ the direction perpendicular to the radius OE, at the point at which the jet leaves the surface or vane, x,u,v_r,a and γ designating the same quantities as before and u' the velocity of the vane at E.

Then, according to the conditions of maximum effect, the jet should leave the van at E, in a direction opposite to u', the direction and velocity of the point of departure E, around the center, and with a velocity relatively to the vane $(-u')$.

If BC$=v_1$ is the original velocity of the jet with reference to the earth, then the absolute maximum work will be

$$M\frac{v_1^2}{2}.$$

Since the velocity of the vane at B is $u=ar$, and the velocity at E is $u'=ar'$: a being the angular velocity of the motion; and, since for the absolute maximum work, $v_r=$DC must be equal to u'; in the triangle BMC, the distance BD$=u$ and DC$=v_r=u'$ must be to each other as r to r'; r being the radius OB and r'— the radius OE.

The work of impulse will be, as before, Mxu, since the relative velocity in the direction of the motion (*i. e.*, perpendicular to the radius) is all destroyed at B.

The work of *reaction* of the jet in moving from B to E and passing off at E in a direction opposite to u', will be $Mv_r u''$; u'' being a velocity of motion due to a length of radius between r and r', and the total work, supposing at first, $\gamma=o$ will be

$$W=M(xu+v_r u'').$$

For the absolute maximum or

$$W=M\frac{v_1^2}{2}$$

and $E=1$ we must have from the mechanical conditions $v_r=u'$ and this expression becomes

$$W=M(xu+u'u'')=M\frac{v_1^2}{2}.$$

The value of $u'u''$ which gives the maximum may be found from the following considerations. If the vane had a velocity at all points equal to u, then this term $u'u''$ would be u^2 as in the case first discussed, and if its velocity at all points was u', this term would be u'^2. The true value when the vane revolves around an axis O, will be a mean between u^2 and u'^2, or $\frac{u^2+u'^2}{2}$—and

$$W_{max.}=M\left(xu+\frac{u^2+u'^2}{2}\right)=M\frac{v_1^2}{2}$$

and

$$E=\frac{\frac{Mv_1^2}{2}}{\frac{Mv_1^2}{2}}=1.$$

Another method of proof is as follows:

In the triangle BMC.

$$(x+u)^2+v_r^2-x^2=v_1^2$$

or $$(x+u)^2+u'^2-x^2=v_1^2$$

developing and cancelling

$$xu+\frac{u^2+u'^2}{2}=\frac{v_1^2}{2}$$

Since $u=ar$

" $u'=ar'$ $\therefore$ $u=u'\frac{r}{r'}$

and

$$\frac{u^2+u'^2}{2}=\frac{u'^2\frac{r^2}{r'^2}+u'^2}{2}$$

$$=\frac{u'^2}{2}\left(1+\frac{r^2}{r'^2}\right)$$

$$=u'^2\left(\frac{r'^2+r^2}{2r'^2}\right)$$

$$=u'ar'\left(\frac{r'^2+r^2}{2r'^2}\right)$$

$$=u'a\frac{r'^2+r^2}{2r'}$$

Hence $$u''=u'.a.\frac{r'^2+r^2}{2r'}$$

and $\frac{r'^2+r^2}{2r'}$ is the radius or lever arm at which the velocity is u''.

The value of u which gives this maximum will be

$$u=u'\frac{r}{r'}$$

The formula for the absolute maximum work then becomes

$$W_{max.}=M\left(xu+\frac{u^2+u'^2}{2}\right)=M\frac{v_1^2}{2}$$

and the efficiency

$$E=1$$

This formula may, from what precedes be put under the form

$$W_{max.}=M\left\{xu+\frac{u'^2}{2}\left(1+\frac{r^2}{r'^2}\right)\right\}=M\frac{v_1^2}{2}$$

In Fig. (5) when the center of motion is O, the stream flows outwardly from the center and r' is greater than r.

If the center of motion were at O', the stream would flow inwardly, or towards the center, and r would be greater than r'.

If $r=r'$, then $u=u'$ and the formula becomes

$$W_{max.}=M\left(xu+u^2\right)=M\frac{v_1^2}{2}$$

as before found.

If the angle γ is not zero; but the relative velocities such as would otherwise give the absolute maximum of work, the real work performed and the efficiency may be found as in the first case.

The lost work will be, as before $M\frac{v_r^2\sin.^2\gamma}{2}$ and the total work

$$W=M\left(xu+\frac{u^2+u'^2}{2}-\frac{V_r^2\sin.^2\gamma}{2}\right)$$

$$=M\left(\frac{v_1^2}{2}-\frac{v_r^2\sin.^2\gamma}{2}\right)$$

and for the efficiency

$$E=\frac{M\left(\frac{v_1^2}{2}-\frac{v_r^2\sin^2\gamma}{2}\right)}{M\frac{v_1^2}{2}}=1-\frac{v_r^2\sin.^2\gamma}{v_1^2}$$

The relation between v_r and v_1 may be found from the triangle BMC, Fig. 6.

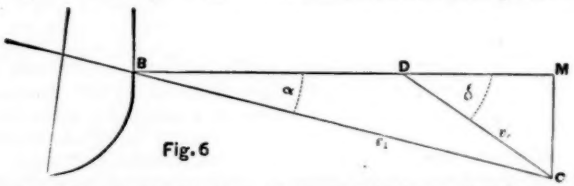


Fig. 6

If the angle CDM be designated by δ, then

$$v_1\sin.a=v_r\sin.\delta$$

and $$\frac{v_r}{v_1}=\frac{\sin.\ a}{\sin.\ \delta}$$

then $$E=1-\frac{\sin.^2a}{\sin.^2\delta}\sin.^2\gamma$$

From this expression it appears as before that the efficiency is increased by making a and γ small.

Variations of work for small variations of u and v_r.

It is evident from the conditions of this problem that a variation of the angular velocity of motion, will cause

simultaneous and proportional changes in the velocities u and u' and in the expression for work

$$W=M(xu+v_r u) \quad . \quad . \quad . \quad (\gamma=o)$$

The work will vary as the sum of the squares of the ordinates DO and DO′ (or as $\overline{DO}^2+\overline{DO'}^2$) of the same circles described on the diameters $\overline{x+u}$ or BM and $\overline{v_r+u''}$: u'' being a quantity whose value lies between u and v_r.

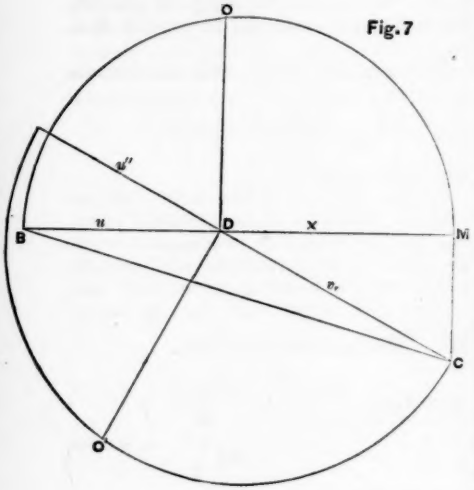


Fig. 7

The squares of these ordinates will vary very slowly near the values, which make the work an absolute maximum, since the ordinates in this case lie near the centers of the semicircles.

The expression

$$W_{max.}=M\left(xu+\frac{u^2+u'^2}{2}\right)$$

or

$$W_{max.}=M\left(xu+\frac{u'^2}{2}\ 1+\frac{r^2}{r'^2}\right)$$

is applicable to the three principle types of turbine wheels, by giving to r and r' their proper *relative* values.

For outward flow wheels $r<r'$
For inward flow wheels $r>r'$
For parallel flow wheels $r=r'$

The diagram (Fig. 8) shows that since the ratio of r to r' is the same as that of u to u', that for the same angle a, the velocity of maximum efficiency (BD′$=u$) will be greater for the inward flow than for the outward flow (BD$=u$) and that for each type of wheel with the same angle of incidence, the velocity of maximum efficiency will have a particular value, viz: for parallel flow u will equal v_r the relative velocity, $\left(=\frac{v_1}{2\ \cos.\ a}\right)$ for outward flow wheels, u will be less than v_r and for inward flow, u will be greater than v_r

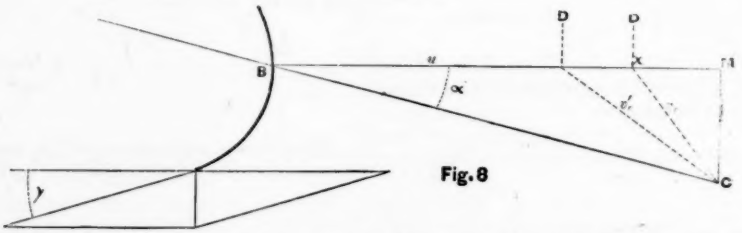


Fig. 8

We have now to discuss particular cases of the general expression for work

$$W=M(xu+v_r u'')$$

as applied to different classes or types of water-wheels. Supposing in all cases $\gamma=o$.

IMPACT WHEELS.

This class includes all water-wheels in which the fluid stream acts by impulse, the blades or floats being plane and no reaction taking place from the curvature of the blades after impact.

The axes of such wheels may be vertical, with a single stream or several streams impinging upon plane floats; or the axis may be horizontal as in the case of the old undershot wheel.

To apply the formula to this class, it is only necessary to make the second term zero. The formula will then become $W=Mxu$, and since $x=v_1\ \cos.\ a-u$

$$W=M(v_1\ \cos.\ a-u)u.$$

This expression is a maximum when $v_r \cos.\ a-u=u$ or

$$u = \frac{v_1 \cos. a}{2}$$

This is the velocity of greatest efficiency, and the efficiency is in all cases

$$E = \frac{W}{\frac{Mv_1^2}{2}} = \frac{Mu^2}{\frac{Mv_1^2}{2}} = \frac{M\frac{v_1^2 \cos.^2 a}{4}}{M\frac{v_1^2}{2}}$$

or

$$E = \frac{\cos.^2 a}{2}.$$

If a is zero, then $u = \frac{v_1}{2}$ is the velocity of greatest efficiency, and $E = \frac{1}{2}$. The usual efficiency of these wheels in practice, is only about $\frac{1}{3}$.

REACTION WHEELS.

A reaction wheel is one in which there is no effect from impulse; or in the general expression

$$W = M(xu + v_r u')$$

it is necessary to make $x = o$. The expression then becomes

$$W = Mv_r u''$$

In these wheels, the deviation of the stream caused by the curvature of the blades is the cause of the energy exerted and several distinct cases may arise.

First, suppose the stream of flow in a direction parallel to the axis of revolution as in the Jonval Wheel; that is, suppose it to be a parallel flow reaction wheel.

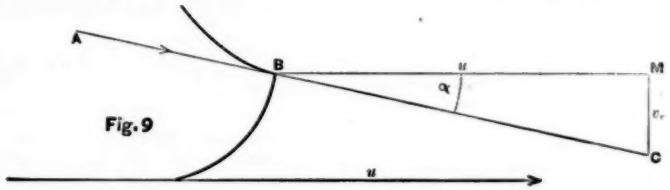


Fig. 9

Then since $x = o$ (Fig. 9) $u = v_1 \cos. x$ and the expression becomes

$$W = Mv_r v_1 \cos. a$$

But the same construction shows that $v_r = MC = v_1 \sin. a$ and the value of W becomes

$$W = Mv_1^2 \sin. a \cos. a$$

For this expression there is no absolute maximum except in one particular case, *i.e.*, for $a = 45°$ in which case $v_1 \sin. a = v_1 \cos. a$ and $v_1^2 \sin. a \cos. a = v_1^2 \sin.^2 a = \frac{v_1^2}{2}$ and $W = M\frac{v_1^2}{2}$

$$E = 1$$

It follows from this that if in the parallel flow wheel, the principle is insisted on that the water must enter without shock, or in the language of Bresse, if "it is necessary that at the point B, (Fig. 8) the water shall be directed tangentially to the floats" the latter being radial at the point of entrance of the water, the wheel becomes simply a reaction wheel and there is no theoretical absolute maximum except for the particular angle $a = 45°$.

Bresse, in discussing the problem for the Fourneyron and parallel flow turbines, deduces nine equations involving sixteen unknown quantities, which involve the necessary conditions of a perfect wheel. A discussion applicable only upon the particular conditions which he presents, but not applicable to American Turbine wheels as now constructed and employed. Nevertheless his discussions which have been translated for use in this country are entitled *Turbine Wheels*, and there is no indication given that they are not applicable to all Turbines.

Second.—If under the supposition that there is no impulsive action of the water, we suppose the stream to flow outwardly from the axis. The work performed will be as before for $x = o$

$$W = Mv_r u'$$

or putting for $v_r u''$ in the general case $\frac{u'^2}{2}\left(1 + \frac{r^2}{r'^2}\right)$ we have

$$W = M\frac{u'^2}{2}\left(1 + \frac{r^2}{r'^2}\right).$$

This applies to the case where the water enters tangentially to the floats, which are radial in direction as at B Fig. 7 or 8, and for which the value of $u = v_1 \cos. a$.

The condition is thus fulfilled as expressed by Rankine, that "the whirling velocity of the water, when it enters the wheel, must be equal to the tangential velocity of the wheel," a condition appli-

cable, according to this author, to all turbines.

But in this case it is evident from the expression

$$W=M\left\{\frac{u'^2}{2}\left(1+\frac{r^2}{r'^2}\right)\right\}$$

that there is no theoretical maximum and the wheel becomes simply a reaction wheel. For an absolute maximum or an efficiency equal to unity, we must have

$$W_{max.}=M\frac{u'^2}{2}\left(1+\frac{r^2}{r'^2}\right)=M\frac{v_1^2}{2}$$

which can only be the case when $r=r'$ and $u'^2=\frac{v_1^2}{2}$ which takes us back to the particular case of the parallel flow wheel, with an angle $\alpha=45.°$

In the outward and inward flow reaction wheels, *i.e.*, those wheels in which the buckets are radial at the point of entrance of the water, and in which the velocity of the wheel at this point is the same as the component velocity of the water in the same direction, there is no work of impulse, and the wheels become reaction wheels, for which the efficiency cannot be unity, but there will always be a loss of energy due to the ratio $\frac{r}{r'}$.

Application of the general formula for work

$$W=M(xu+v_r\,u'')$$

to the wheel known as Barker's Mill.

In this wheel the water descends in a column H, Fig. 10, which is at the same

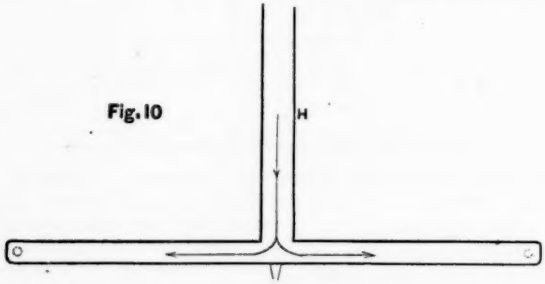


Fig. 10

time the axis of motion, and flows outwardly to the ends of two tubular arms where it issues from two orifices on opposite sides of the arms.

As the water enters the arms without impulse, the term Mxu in this formula will be zero and the expression for work will be

$$W=Mv_r\,u''$$

u'' will be the circular velocity of the end of the arm at the point where the water issues, v_r, being the relative velocity of the arm and fluid at the same point, may be found thus. In the plan of the wheel, Fig. 11, let v_1 be the velocity due to the head. A particle at the end of the arm is subjected to two velocities, one v_1 due to the head and another u'' due to the rotation of the arm. Its actual velocity with reference to the earth, will be represented by the diagonal of the rectangular parallelogram constructed on these

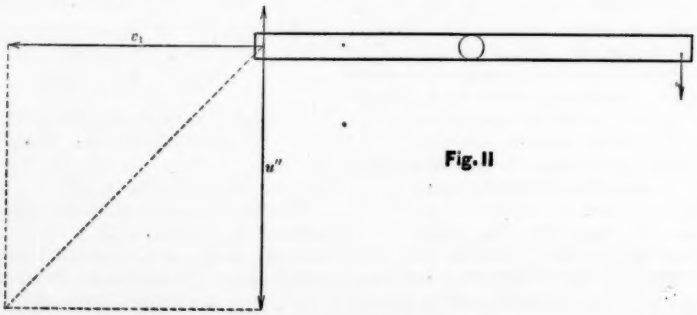


Fig. 11

two lines, and will be expressed analytically by

$$\sqrt{v_1^2 + u''^2}$$

This will be the velocity with reference to the earth, with which the jet will issue from the arm in a direction opposite to its motion; but the arm has a velocity $u'' = ar$ (a being the angular velocity and r the radius of the arm) and the relative velocity with reference to the arm will be

$$v_r = \sqrt{v_1^2 + u''^2} - u''$$

and the work performed will be found by multiplying this by u'' and by M, the mass of water which issues in a unit of time or

$$W = Mv_r\, u'' = Mu''(\sqrt{v_1^2 + u''^2} - u'')$$

an expression which has no maximum.

In the foregoing discussion, no account has been taken of the resistances which arise from friction, or loss of energy from contractions, bends and other causes which influence the flow through narrow passages, like the spaces between the guide blades and the wheel buckets or floats. These influences will, of course, affect the efficiency of all wheels, and also the velocity of maximum efficiency and, of course, in no actual case can an efficiency of unity be attained. The general theory is not, however, altered on this account; and inasmuch as it is impossible to separate the influences referred to from each other, or to ascertain the values of co-efficients which might be applied, the only means of ascertaining how near the performance of any wheel approaches the maximum efficiency is by a practical test.

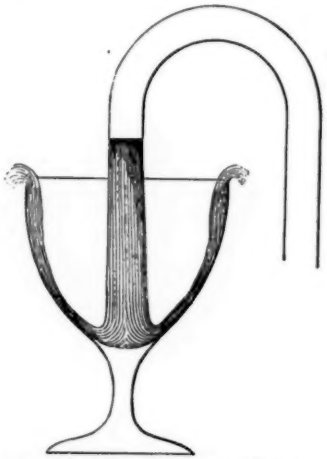
From Francis' experiments it appears that the velocity of maximum efficiency is somewhat greater than that which would be given by theory when friction is not considered.

It has been shown that a considerable change in velocity of a wheel when it runs at a speed giving its maximum efficiency, influences but little, the work performed.

A change in velocity in the Tremont turbine from .48 to .68 of the velocity due to the fall, affected the efficiency by only two per cent., and Francis recommends a velocity of .56 of the velocity due to the fall, as the best in practice, which is in excess of the theoretical velocity of greatest efficiency, when friction is not considered.

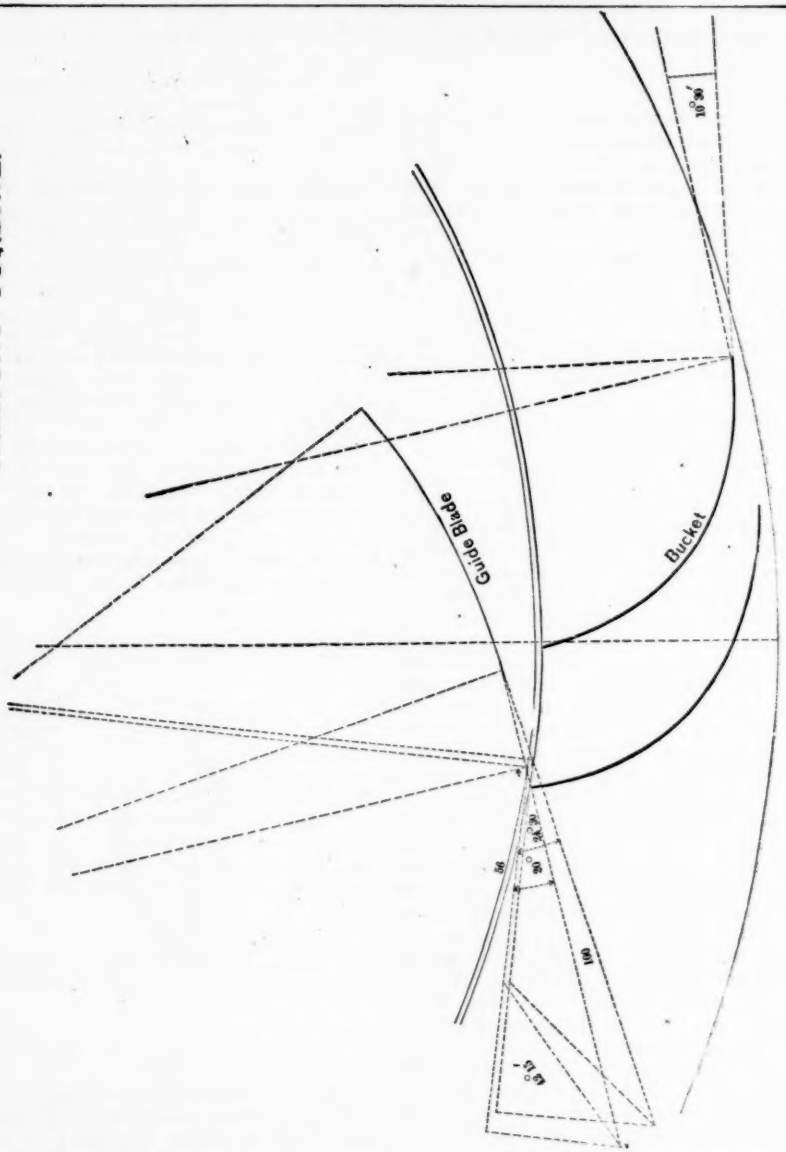
The following diagram of the Tremont turbine and Francis' center-vent, or inward flow turbine, exhibit the particulars of these wheels: 1st, in the small ratio $\frac{r}{r'}$, of the outer and inner radii and the small angles of incidence and departure of the water; and when the velocity of the wheels which give the maximum efficiency is considered, it appears that the velocities of the wheels, at the point where they receive the entering water, is much less than the tangential component of the entering water, thus causing a large part of the work to be done by impulse.

An unnecessary importance seems to have been attached to the idea that a stream of water, to produce its best effect upon a vane, or float, must glide upon the latter in a tangential direction. The following simple experiment may be made by any one who wishes to be convinced of this.



Place a goblet under an ordinary goose neck spout, from which a clear and transparent stream issues, impinging on the bottom of the goblet. If the velocity of the stream is sufficient, the water will rise up and flow over the edge of the goblet at all points in a clear, unbroken sheet, the liquid threads being continuous and unbroken, and the whole remaining as transparent as the glass itself. A more satisfactory experiment may, per-

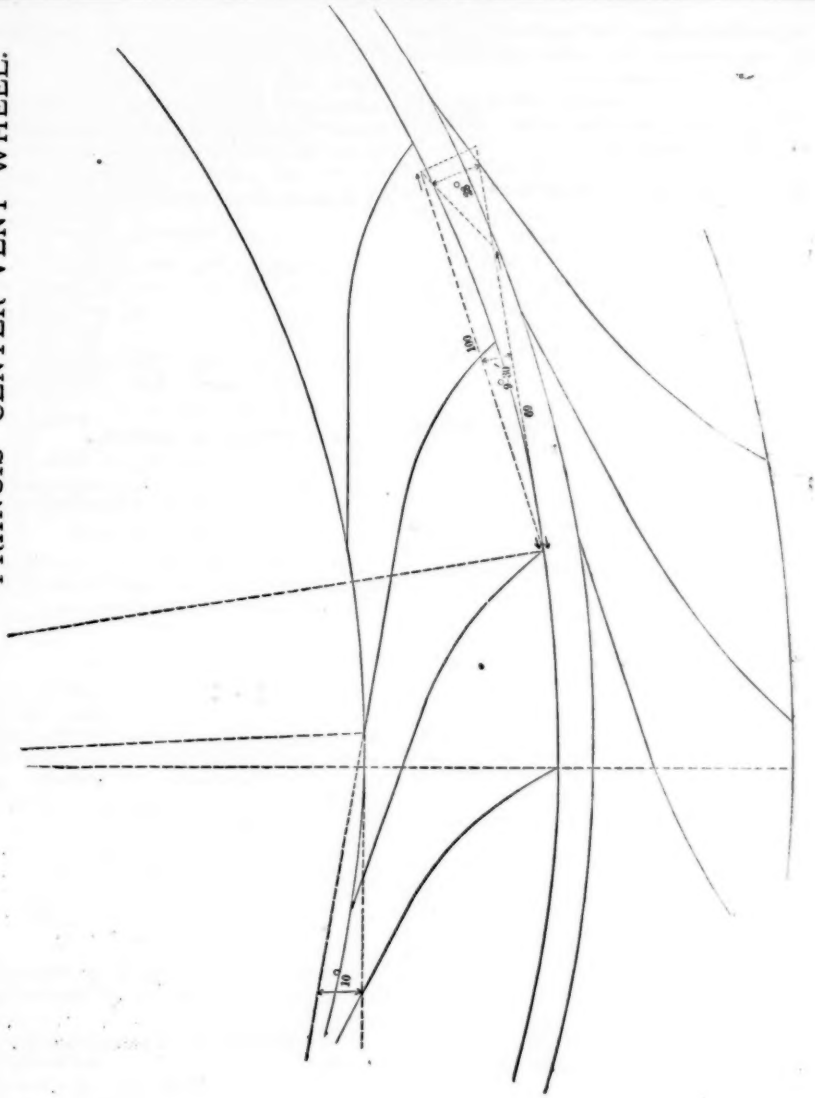
TREMONT TURBINE.



haps, be made by inverting the goblet and allowing the stream to enter from below; more satisfactory, because considerable head is required in the first case. This experiment shows that the disturbances in the stream, from impact, which have been considered of so much importance, do not exist in this, the most unfavorable case, where the impact is normal.

The Poncelet wheel, a vertical water wheel, the construction of which is familiar to all engineers and students, is one in which a principal virtue is supposed to

FRANCIS' CENTER VENT WHEEL.



exist in a curved form of the buckets—such that the water enters the wheel "without shock."

Weisbach states that "to obtain as great an effect as possible with one of Poncelet's wheels, it is necessary that the water should enter the wheel without shock." And it has been considered that one object of the peculiar construction and arrangement of Poncelet was to attain this particular end; whereas a little consideration shows that the curvature of the blade and the peculiarities of construction were designed for quite another purpose.

In fact, the analysis would be quite as

complete for this wheel as far as impulse is concerned, if the water impinged directly upon a plane float.

It is only to obtain the effect of reaction by the returning water that the curvature is necessary.

In order to find the actual value of the efficiency as deduced from the formula

$$E=1-\frac{\sin.^2\alpha}{\sin.^2\delta}\sin.^2\gamma$$

we have only to substitute values for α, δ and γ. Taking, for example, $\alpha=22°$, $\gamma=11°$ and $\delta=46°$ the values employed in the Tremont Turbine, substituting and solving we have

$$E=.9902$$

For Francis' Center Vent Wheel $\alpha=9°30'$ $\gamma=10°$ and $\delta=28°$ which give for the efficiency

$$E=.9962$$

These results show a theoretical efficiency practically equal to unity, when friction and other disturbing resistances are not considered. The angles of incidence and departure being so small as to have but a slight influence in lessening the efficiency when the proper theoretical velocity is given to the wheels.

The angles of incidence of the Swain wheel are about 12° only, and the angle of departure of the inward flow wheels now made is not much greater. It is not surprising, therefore, that a common working efficiency of 84 per cent. should be attained.

The efficiency of an impulse and reaction turbine, properly consructed and run with the proper velocity, being thus practically unity, or 99 per cent., when loss of energy from friction, and other causes of loss of energy of a similar character, are not considered the greatest possible efficiency in practice.

Estimating the co-efficient of velocity of discharge from the guide blades at ,97 the loss of energy due to this loss of velocity will be, $[(\frac{1}{97})^2-1]\frac{Mv_1^2}{2}$ equal $.06\frac{Mv_1^2}{2}$; and supposing the friction in the wheel to consume the same amount of energy, the total loss will be $,12\frac{Mv_1^2}{2}$.

If we suppose the friction of the pivots on which the wheel runs, and the collars of the axis to be in the most favorable case $,03\frac{Mv_1^2}{2}$ we have a total of $,15\frac{Mv_1^2}{2}$ which with the loss of .01 due to the obliquity of the angles α and γ will give a total of 16 per cent. of the energy due to the fall, leaving a practical efficiency of 84 per cent. which is the efficiency found by Francis for an ordinary Swain wheel.

RULES FOR DESIGNING WHEELS.

Let D be the diameter of the wheel.

Let α be the angle of incidence of the water with the tangent to the circumference.

Let H be the depth between the crowns at the first circumference or point of entrance.

C a co-efficient of discharge.

A the area of section of entrance of water in the direction of the radius, then the volume of flow will be

$$V=\pi D.\ H.\ v_1 \sin.\ \alpha.\ C.$$

Assuming a diameter D. a co-efficient of discharge C and an angle of incidence α the depth H will be

$$H=\frac{V}{C.\ \pi\ D.\ v_1 \sin.\ \alpha}$$

By experiments on the Tremont turbine in which D=8.25308 feet=sum of the widths of the orifices of discharge across the guide blades, and for the heights of the orifices of discharge H, or depth of wheel=.9314 feet. Francis found C=.624.

Hence his general rule, making $H'=\frac{1}{10}$ of the *outer diameter* D',

$$V=H'.\ D'.\ C\ v_1=0.10\ D'^2 e\sqrt{2gh_1}$$
$$=0.5\ \ D'^2\sqrt{h_1}$$

In which H_1 is the *depth of the wheel at the outer extremities of the buckets* and $H'=\frac{1}{10}D'$.

The simplicity of Francis' formula for the flow

$$V=0.5D'^2\sqrt{h_1}$$

is one of the results of his valuable experiments, giving by a very simple calculation the necessary diameter of an outward flow turbine for a given head h_1 and a given volume of flow V. For a given horse power, $\overline{HP}$ in foot pounds per second, with an efficiency of 75,

$$\overline{HP}=\frac{0.75\times 62.33}{550}\times V h_1$$

and since $V=0.5\,D'^2\sqrt{h_1}$

$$D'=4.85\sqrt{\frac{P}{h_1\sqrt{h_1}}}$$

Formulas which are applicable to outward flow wheels, within the limits assigned by Francis.

To make the formula more general, we have

$$V=.62\ D.H.\pi.v_1\ \sin.\ a$$
$$=f.D^2\ v_1\ \sin.\ a\ \pi.\ ,62$$

f being ratio of D to H.

Supposing V to be the actual volume of flow per second, and the wheel to utilize .75% of the fall h, multiplied by the weight, then we have the horse-power per second,

$$HP=\frac{0.75\ .(62\ .43)}{550}\,Vh_1\ \ldots\ (a)$$

and substituting for V its value

$$V=f.D^2.v_1\ \sin.\ a\ .62\ \pi$$
$$=f.D^2.v_1\ \sin.\ a\ 1.95$$

we have

$$HP=\left(\frac{0.75\times 62.43}{550}\times 1.95\right)f.D^2.v_1\sin.a\,h_1$$

or

$$HP=\left(\frac{0.75\times 62.43\times 1.956}{550}\right)$$
$$f.D^2.\ \sqrt{2gh_1}\ \sin.\ a\ h_1$$

$$HP=\left(\frac{0.75\times 62.43\times 1.956\times 8\ .03}{550}\right)$$
$$f.D.^2\sin.\ a\,h_1\ \sqrt{h_1}$$

$$HP=1.33\,f.D^2.\ \sin.\ a\ h_1\ \sqrt{h_1}$$

$$D=\sqrt{\frac{HP}{1.33\,f.\ \sin.\ a\,h_1\ \sqrt{h_1}}}\ \ \ldots\ \ldots\ (b)$$

f being the ratio of diameter to depth, H.

Having found the diameter D (the diameter of that circumference on which the water first impinges) whether for outward or inward flow, the second diameter may be found from the relation of the radii shown in the diagrams which have been given.

The depth $H=f.D.$ of the wheel should be somewhat greater at the discharging diameter D', in order to make allowance for the greater space occupied on the circumference by the oblique positions of the extremities of the buckets.

The angular velocity of the wheel being $2\pi N$ in which N is the number of revolutions per second. The tangential velocity will be

$$u=2\pi Nv$$

For parallel flow wheels

$$V=C.A.v_1\ \sin.\ a$$
$$=C.\pi(r^2-r^2)\ \sin.a$$

C being a co-efficient of discharge and A the area of the annular space through which the water flows, equal to $\pi(r'^2-r^2)$.

If D be the mean diameter and f the ratio of D to $(r'-r)$ or $f=\frac{r'-r}{D}$ then the formula (b) will apply to parallel flow wheels.

FORM OF BLADES.

For the three typical wheels, the *outward flow*, the *inward flow*, and the *parallel flow*, the path of a particle of water continues in one plane. In the outward and inward flow wheels, the planes of motion of the particles are perpendicular to the axis. In the parallel flow, the planes of the paths are parallel to the axis.

The lips of the buckets at the points of entrance to the wheel are generally straight edges, and for these three wheels are either perpendicular or parallel to the axis of motion.

The discharging edges are lines parallel to the receiving edges and the buckets at the discharging edges have a constant angle γ, along the edge, with the circumference.

In the outward and inward flow wheels the edges are elements of the cylindrical surfaces of the outer and inner circumferences of the wheel, and in the parallel flow, the receiving and discharging edge of the buckets are radial.

Combinations of these simple wheels, however, present different conditions; the *inward* and *downward* flow wheels represented by the Swain and Risdon, for example, and the *outward* and *downward* flow represented by the Excelsior wheel.

The general principles to be kept in view for all buckets, are that the channels between the buckets shall not have abrupt changes in direction; that they shall be as short as possible; that the curvature of the buckets shall be continuous; that the discharging edges of the buckets shall have a uniform discharging angle, and that the cross-section of the

channels between the buckets shall be uniform throughout. It is especially important that the water should leave the guide blades and enter the wheel in clear, transparent streams, without contraction, in order that that these streams may continue unbroken through the wheel to the point of discharge.

SEWAGE AND IRRIGATION WORKS IN GERMANY.

From "The Builder."

The question of sewage irrigation, after having been allowed to recede somewhat into the background for some time, is again attracting public attention to a more than ordinary degree. It is a subject which has repeatedly given rise to lively controversy in England. On that account, a reference to what other countries have been doing, and are still doing, in the matter will be of interest. The steps taken in Germany, with that view especially, will throw additional light on the knotty point whether sewage irrigation "pays." Authorities at home disagree on the financial aspect of such undertakings, but it has been proved that better crops can be grown with sewage alone than could be grown under tillage, costing sometimes as much as from 2*l.* to 3*l.* per acre. It has also been shown that if the sewage can be delivered upon land by gravitation close to the sewage works, this mode of disposing of it is more profitable, as well as more consonant with the objects of sewering towns, than discharging it into rivers at some distance below them. Others, again, will have it that the most economical plan to dispose of town sewage is to carry it, if possible bodily, far enough into the open sea that there is no chance of its being brought back again by the tide. Whichever system may be preferred, or whichever may be the most practicable under varying conditions, we think it will be sufficient to show that if irrigation by sewage tends to decrease the expense of removing it from towns, it will be the more economical as well as the safer way of disposing of it. As we have said, and we might cite numerous cases in this country, confirmatory of our view, so much has been proved. For this reason we need not dwell any longer on the question as it affects England, but proceed at once to examine its aspects in Germany. And here, again, it will not be uninteresting to note that the Germans have taken a leaf out of our book: that they are introducing, or have introduced, a system of sewerage into some of their larger cities at least, partly by the help of British engineering and British capital; which is eminently true in the case of Danzig, at one time one of the most unhealthy cities of the Continent, and a city, the purifying of which, was a far more Herculean task than the celebrated cleansing of the Augean stable.

Although the Danzig contract itself was based upon the extravagant reports prevalent in England (about 1865 to 1870) as to financial results of sewage farming, we have it upon the best authority that experience there is such, that similar undertakings, if properly arranged and conducted, may be confidently recommended for safe and satisfactory investments. Indeed, the Danzig undertaking has turned out so well, as we shall presently be able to show, that the municipal authorities of another large city, viz., Breslau, formerly also notorious for its insalubrious condition, with a population of 270,000 souls, have recently concluded a contract with Messrs. J. & A. Aird & Mare, the eminent Berlin firm of engineers, who completed the Danzig works, and have been working the sewage farm there for some time, for laying out the land and undertaking the management of the works and farming for twelve years. Moreover, Berlin, with over a million inhabitants, after having for several years made a most exhaustive series of trials of all the known or projected systems, has finally adopted the irrigation system. The chief engineer of the Berlin municipality, Herr Baurath Hobrecht, states the total cost of the irrigation arrangements, including purchase of the land, buildings, cost of mains from the pumping stations, laying

out the lands, roads, etc., to be 260 thalers per morgen, or about 60*l.* per acre, the morgen being about six-tenths of an acre. The Berlin authorities are completely satisfied with the financial results.

After this general preliminary statement, let us consider the case of Danzig; and as the supply of water is closely connected with the removal of sewage—indeed the latter could not be thought of without, a few remarks on that point will be necessary. The municipality of Danzig resolved as early as 1863 to provide the city with pure drinking-water and a complete system of sewerage. The Danzig waterworks, if a system of wooden pipes liable to pollution from external causes may be called by that name, dated from the time of the German Knights. They only supplied water for general use, however, and drinking water was brought into the city on wagons, and was sold at a high rate. However, with the opening of the new waterworks in 1869, by which pure water was brought a distance of about fifteen miles from reservoirs *circa* 350 ft. above sea level, all this was changed. The daily supply, since the day of their opening, has varied between 245,000 and 400,000 cubic feet. The total cost for executing all the works connected with the undertaking has been not quite 550,000 thalers (£82,000).

The draining system of the city was so defective that it would have been extremely injudicious to discharge into it an increased quantity of used and polluted water. The highly dangerous accumulation of filth of all descriptions in houses and courts, street gutters and cesspools, as well as in the public water reservoirs, called for urgent reform, by introducing a new system of drainage. When, therefore, the question of water supply was once determined, the carrying out of drainage works could be seriously thought of. A contract was concluded with Messrs. Aird for executing them, and detailed plans, on the project as submitted by Herr Barauth Wiebe, of Berlin, were prepared by Mr. Baldwin Latham, C. E. The works were begun in August, 1869, and completed by the end of 1871, at a cost of 700,000 thalers (£105,000).

There remains to consider the more important part of our subject, the disposal of the sewage for irrigation purposes in Danzig. In the absence of a more conveniently situated and better site, the dune district on the coast of the Baltic, between Weichselmünde and Heubude, the property of the city of Danzig, has been selected as the ground for purifying the sewage and utilizing it for agricultural purposes. The land in question was partly covered with firs; about 200 morgens were let for pasturage, at the low rent of 50 thalers.

By the contract of September 13, 1869, Mr. A. Aird acquired the right to use the sewage of Danzig, as well as a grant of 2,000 morgens for irrigation and farming purposes, for a term of thirty years, for which Mr. Aird undertook, for a like period, to keep in perfect repair and working order all the sewers and drains, and the pumping-station. The expenses for this undertaking of Mr. Aird's may be estimated at from 8,000 to 10,000 thalers (£1,500) per annum. At the end of the term, Mr. Aird has to give up possession of the land, with all improvements and plant, without claiming compensation for the capital laid out in the cultivation of the land. Any buildings erected by him he may take down and remove, unless the city of Danzig choose to purchase the same at a valuation. The contract was a risky undertaking, yet, as will be shown by and by, Mr. Aird has rendered by it a great service not only to Danzig but also to other German cities which have undertaken similar works.

The plant at the pumping station includes two steam engines, each sixty-horse power, of which one is at work, as a rule, only from fourteen to eighteen hours a day. During heavy rains, and from May to September, one engine is always at work. In the former case, from 350,000 to 390,000 cubic feet, in the latter, from 525,000 to 550,000 cubic feet, —of which about 175,000 cubic feet are water let in from the river Mottlau, as then the sewage alone would not be sufficient—are daily taken to the area of irrigation. The channel into which the delivery-pipe empties itself lies at the highest point of the dune, feeding from here all the principal irrigation trenches. The latter branch off on both slopes, and give off again into smaller channels. The ground is leveled in accordance with the gradient, the irrigation being affected according as it is wanted, either

by damming up or through furrows. Should there be any surplus sewage, this flows into the sea.

A few words as to the nature of the ground. The dune district in question, it should be stated, is, by its nature, very little suited for agricultural purposes, and for purifying sewage. The sterility of the dune sand is still further increased by being impregnated nearly everywhere with a fine red sand, so-called *Fuchssand* (a kind of red garden gravel). The hilly configuration of the ground, moreover, made the works of leveling rather expensive, the cost of the area brought under cultivation being £11 5*s.* per morgen (£15 15*s.* per acre), but will be much less in the future. In consequence of the capillarity of the sand, the level of the ground water is very high, and this increases the difficulty of draining. The growth of the shooting and young plants is not unfrequently jeopardised by sand-drifts.

Although dune sand at first permits sewage water to run through it more quickly than is desirable, its power of absorption is still great enough to allow of the development, even in the first year, of a luxurious vegetation of the most exacting plants. In consequence of the property inherent in roots of plants, not only of appropriating substances contained in solution in the ground water, but also of dissolving and absorbing matter coming into contact with the finest root-fibres, luxuriousness of vegetation contributes no less to purifying of sewage water than does the power of absorption of the soil. But the latter also steadily and speedily increases, consequent upon the rapidly progressing formation of humus, to which the organic substances suspended in sewage water, and being deposited in the sand, as well as the decomposing remains of vegetable substances produced, contribute equally largely.

The filtrated water, if it does not come into contact with the red sand, and is not colored reddish by it, is quite clear, free from smell, and tasteless. The purifying process has been complete. The filtrated water has been tested in vain by chemical analysis for the complicated chemical elements of a urinary or fœcal nature. They have, as far as they have not been absorbed by the soil or assimilated by the roots of plants, experienced a lasting change of substance, and have been oxydized to carbonic acid, nitric acid, ulmic acid, butyric acid, etc., by the oxygen of the air contained in the soil and continually replaced.

Of the dune area intended for agricultural purposes, there were in cultivation, in 1872, 30 morgens; in 1873, 120 morgens; in 1874, 250 morgens. These operations had, to some extent, the character of experiments, which were to form sound foundations for future cultivation. There not being a market for fresh-cut grass, and haymaking interfering with the development of the after-growth, the cultivation of grass is considerably confined, and that of beet-root, turnips, maize, oil-seeds, cereals, vegetables, also tobacco, carried on. In 1876, 500 morgens were leveled and prepared for cultivation and tilled as follows:—90 morgens with beets, 34 morgens with colza, 60 morgens (on which that year for the first time oats were grown) with rape-seed, 50 morgens with buckwheat, 12 morgens with barley, 12 morgens with tobacco and maize, 15 morgens (on which rape-seed and flax had stood) with late turnips, 1½ morgen with hemp, 1 morgen with caraway-seed, 50 morgens with Timothy grass and clover (sown end of June and beginning of August), 5 morgens with vegetables; 170 morgens of freshly leveled land were reserved the same year (1876) for planting next spring with beets and other summer crops.

The irrigation, as already stated, was begun on a small scale in 1872 after—at the end of 1871—the sewerage system had been completed. A beginning was made, as in England, with Italian rye-grass. An area of eight morgens, first laid out, was sown on May 1, with rye-grass; the first cut was taken on June 12, of a length of 18½ in.; the second on July 5, of 19½ in.; the third, on July 30, of 20 in.; the fourth, on September 5, of 20½ in.; the fifth, on November 1, of 28½ in. The second area, of 6¼ morgens, was sown on June 7, and gave four harvests—on July 8, August 7, September 23, and November 3—of similar lengths. The third area, of 4 morgens, was sown on August 14, and gave in October, a cut of 24 in. long. In 1872, the grass grew at the rate of 1 in. in twenty-four hours.

Those few figures show the extraordinary effect of sewage irrigation upon sterile sand; indeed, the growth of grass the following year was still greater. Although rye-grass is the best for irrigation, because it not only withdraws, by its fine roots, most surely the organic matter contained in sewage (according to recent observations even animal substances direct, like insectivorous plants), but also favors, by the intertwining of its roots, the filtration of water, it was found necessary to attempt also other products of the field, as no buyers could be found for the great quantity of grass obtained, and there was not a large stock of cattle in the neighborhood. Mr. Aird had to decide whether he was to acquire a large farming stock and become a cattle-breeder, or whether he was to proceed to utilize the irrigated are a in another way. Beets, wheat, rye, rapeseed, mustard, and tobacco, were grown; next, vegetables were attempted; and, finally, also flowers—the latter for their seeds. Besides the area under cultivation by him direct in 1876, 36 morgens were devoted to vegetables and flowers, and 244 morgens let out on leases. The farmers grew, especially Swedes, summer corn, and tobacco (18 morgens). The cultivation of the latter is said to be very remunerative, and the tobacco, if not a Havana leaf, to be certainly of a good flavor.

But not only the dimensions of the area cultivated have increased, the yield has become better paying, even handsome. A proof of this is, that already half of the irrigation area is let. In 1876, from 15 to 20 thalers were paid per morgen; in 1877, the offers went up as high as 24 thalers per morgen, or about £5 per acre, and this for soil formerly dune sand. Those rents are explained by looking over the yields as given in the table below:—

Cereals, &c.	Seed sown.	Yield of Corn.	Straw.
	Scheffel.	Scheffel.	Cwt.
Winter wheat.....	1.20	15½	22½
Winter rye.....	1.10	12	29
Summer rye.....	1.35	9	12
Barley.....	1.40	11½	12
Oats.....	2.00	21½	16
Winter colza.....	1.40	13½	—
Winter rapeseed...	1.10	14	—
Summer rapeseed..	1.60	9½	—

In 1876, from 200 cwt. to 320 cwt. of swedes, according to the age of the irrigated field, were obtained per morgen. Tobacco yielded about 13½ cwt. The cereals grown were good, partly excellent (rye, oats, summer rape), and sold readily at market rates, as well as the straw, which was very healthy, Swedes fetched in 1876, from 10*d.* to 11*d.* per cwt.; tobacco £1, 11*s.* 6*d.* per cwt. The yield of hay was in the second year (1873) in the area on which Timothy and clover were grown from 26 cwt. to 27 cwt. per morgen, and sold at 3*s.* 9*d.* to 4*s.* per cwt. The area planted with vegetables, and flowers for seed, yielded 110 thaler brutto per morgen. Mustard gave 5½ cwt. per morgen.

An important question arose as to how the irrigation system would work in winter. Danzig is much exposed to the tempering effect of the sea, but still the winters are very severe. But since the beginning of the irrigation, the latter has been carried on throughout the winter without difficulty. The sewage arrives on the field even if the cold weather is continuous; at most a thin ice crust is formed, below which it runs and sinks into the ground. As the growth is then at a standstill, even of rye-grass, it will not be frozen out. Newly prepared ground is principally irrigated in winter, so that it receives a thorough manuring then.

The consumption of sewage-water in summer is already so large that additional water is let into the drains from the river Mottlau. The large quantities of water which are thus daily brought on to the irrigation area must, of course, find a natural outflow. This is, in the first place, the ground-water, the level of which is from 3 ft. to 5 ft. under the surface. The filtrated water collects in small ditches, which lead into larger channels, emptying directly or indirectly into the Vistula.

Of course, we might give detailed statistics of the indirect benefit conferred by the the drainage works on the health of Danzig. But this not being within the scope of the present inquiry, though an important consideration in all sewerage undertakings, we need only remark that they are most striking. To mention only one instance, frequently the cause of great mortality, that of typhus fever, gen-

erally the result of bad drainage. In 1868 the deaths from that disease were eighty-nine; in 1876, they were only twelve.

The above statement will give our readers a fair idea of the Danzig works, and the prospects of similar undertakings in Germany. We may, however, point to the case of Danzig as one in which the change from the formerly prevailing abominable system—system is almost too good a word for a state of matters where everything was worse than chaos,—to a well-regulated arrangement of sewerage and sewage irrigation, coupled with a bountiful supply of pure water, has been most beneficial.

Altogether, the results have been astonishing. On the one hand, a desolate moving dune district has been changed, as if by the wand of an enchanter, into fertile fields and garden land. On the other, a city formerly the permanent stronghold of filth and disease, the soil of which was reeking with unclean accumulations, to which additions were daily being made—its inhabitants breathing the noxious effluvia with such a combination of nastiness produces, and drinking poisonous water,—has been transformed into a healthy city. What is more remarkable, the change has been effected at a comparatively moderate cost. Indeed, the expense must be looked upon as small when we consider what an immense amount of good has been done.

We now proceed to consider the case of the German capital. The drainage of Berlin has been a most difficult and complicated subject for the engineer. Until 1873,—when the necessity for doing something in the matter of the general drainage of their city was recognized by the corporation, and the work of canalization commenced,—the capital was without any properly regulated system of drains and sewers. It is true a number of schemes had been proposed, and for various reasons, some of a technical, others of a financial nature, rejected; the earliest proposal dating from 1816.

About twelve years ago a plan for the drainage of Berlin was submitted to the authorities by Herr Geheim-Oberbaurath Wiebe. In this, one system of sewers for the whole city was projected, with a main outlet in the river Spree near Charlottenburg. Berlin being very flat, and only a few feet above the level of the river, the execution of this project,—a main sewer necessarily falling from end to end of the city (with provision for future extension in all directions),—the subsoil being running sand, permitting water to percolate in large quantities, would have been attended with great difficulties and enormous expense, and was consequently also rejected.

The project of Herr Baurath Hobrecht, adopted in 1873, and now being carried out, however, is based upon what is called in Germany the "radial" system—a system eminently suited to the special circumstances of Berlin, besides preventing the pollution of the river, the sewage being ultimately used for irrigation purposes. There are five systems of sewers, each with a pumping-station, and each starting from the centre of the city and running in a different direction.

The many advantages of this plan are at once apparent. The main sewers are, as a great depth is in no case necessary, comparatively easy to execute; the branch sewers in proportion. As any extensions can only be necessary in the periphery, the works in the central (built-upon) area have an unusually permanent character, and are never likely to be subject to alteration. The extensions necessary can easily be connected without any interruption to the trunk systems. The height the sewage has to be pumped is much reduced. It is undoubtedly also an advantage that the authorities are in no way bound to any one particular site or direction as to irrigation areas. Besides these advantages, the dangers of serious flooding, accidents, disturbances, etc., are greatly lessened by this division and distribution.

The works in Berlin are approachiug completion. Already two radial systems are working, so far as pumping operations are concerned, and already for two seasons irrigation has been carried on, and with most satisfactory results. It is to be hoped that the Metropolitan Board of Works will one day take courage, and address to the municipal authorities of Berlin an official inquiry on the subject.

The irrigation area for the three systems on the south side of the Spree is distant from the various pumping-stations about eight miles. In order to reach the fields by gravitation, the sewage has to

be lifted at the stations nearly 65 ft. In the middle of this year the amount of sewage passed through the pumping-station of the radial system No. III. was nearly 240,000,000 gallons per month, which is only about the sixth part of the maximum capacity of that station.

The irrigation farm of Osdorf comprises, in round numbers, about 2,000 acres, of which at present about 500 acres have been adapted for irrigation. Of this area, 225 acres are grass land, 150 acres are used for market-gardening, while 125 acres are covered by shallow reservoirs, serving for storing provisionally the sewage during winter, should severe weather involve a stoppage of the irrigation. The irrigation ground is undulating, rising in some instances 40 ft. The delivery-pipe is carried to nearly all higher-lying points, where there are outflow-holes with ordinary slides. The steeper slopes are covered with Italian rye-grass, while the more level ground is used for vegetable-growing, and for this purpose formed into terraces, with narrow raised beds, divided by trenches, from which the water penetrates laterally to the roots. The paths between the fields are planted with fruit-trees, the luxurious growth of which, combined with the even more than luxurious look of the fields, imparts a most satisfactory impression.

The irrigation area at Osdorf has been acquired at an expense, including the laying out of the farm, of £125,000 (over £60 an acre). The total cost of the draining and sewerage of Berlin is estimated at £2,000,000. There will be two sewage-farms on the north side of the river Spree, when the drainage of that part of the city is once in working order.

Strange to say, the project was at first looked upon with most unfavorable eyes by the rich as well as the poor of the German capital; but what is stranger still, the municipal authorities of Berlin as those of other German cities where sewage and irrigation works have been projected, met with but scant encouragement from the Government. However, the beneficial effects are already appearing, in a direction, too, in which the poorer classes of Berlin have hitherto been great sufferers. Leaving out of the question the improvement in the sanitary condition of the city,—which, of course, is by itself a highly important factor,—the Berliners are already reaping substantial advantages. The success of market-gardening on the communal irrigation farm, the excellent quality of the vegetables of all descriptions grown there, has sent down the prices of those indispensable commodities wonderfully. "When once the works are a few years older, when a little more experience has put them more in working order, especially when gardeners practised in the systematic cultivation of market-gardens have been engaged," writes us a correspondent intimately acquainted with Berlin, "the Berliners will not be long in turning enthusiastic defenders of the welcome little stranger, formerly very much ill used, which has brought them clean streets without open gutters, pure air, salubrious water-closets and courts, and good vegetables and milk, which will now also be within the reach of the poor man."

A few words in conclusion as regards sewage and irrigation works, with which waterworks are closely connected, in Germany. People in that country are gradually becoming alive to the fact that the health of towns may be improved by the supply of pure air and pure water. Witness the numerous works undertaken with this view, not only in the larger cities, but also in smaller towns, the mere mention of which would make up a very respectable list. Of the more populous cities, besides the two which form the subject of this article, the cases of Breslau and Munich may be cited. As regards Breslau, the sewer system, on which Messrs. Aird, of Berlin, have been engaged during the last few years, will be completed in 7879. Sewage irrigation is to come into operation there on the completion of the works. The lands are already being prepared for irrrigation.

With respect to Munich, plans for the sewerage have been prepared by J. Gordon, C. E. (formerly of Carlisle). The execution of those plans involves an estimated expenditure of £734,000, and they are now being revised by Herr Geheimrath Wiebe. A deputation of the Munich magistracy recently spent some days in Danzig, inspecting the works there and visiting the farm. Departing, they expressed the most thorough approval of the system and the arrangements.

REPORTS OF ENGINEERING SOCIETIES.

AMERICAN SOCIETY OF CIVIL ENGINEERS.—The last numbers of Transactions are well filled with important matter. The following papers are published with abundant illustrations:

No. 166. Reminiscences and Experiences of Early Engineering Operations on Railroads, with especial reference to steep inclines. By W. Milnor Roberts.

Discussions on Inclined Planes for Railroads. By O. Chanute and by Wm. H. Paine.

No. 167. Distribution of Rain Fall, October 3d and 4th, 1869 By James B. Francis.

No. 168. The Gauging of Streams, By Clemens Herschel.

No. 169. Dangers Threatening the Navigation of the Mississippi River, and the Reclamation of Alluvial Lands. By B. M. Harrod.

No. 179. Brick Arches for Large Sewers. By R. Hering.

No. 180. Fall of the Western Arched Approach to South Street Bridge in Philadelphia. By D. McN. Stauffer.

Discussions on above Papers. By E. S. Chesborough, W. Milnor Roberts, F. Collingwood, and R. Hering.

ENGINEERS' CLUB OF PHILADELPHIA.—At the meeting of the Club held January 18th, Mr. D. McN. Stauffer read a paper on "Conical Arches." The eastern approach to the South Street bridge in this city is made up, in part, of a somewhat peculiar piece of arched masonry, which contains certain novel features of useful application at other points.

The center lines of South Street and of the bridge proper intersect at an angle of 33° 25′, necessitating a curve in the approach from the east. To conform in design with the "late lamented" western arched approach, this curve was pierced by three arches. By adopting the conical arch much money was saved in the foundations, in the arch piers themselves, and in the haunching above them. The improvement in appearance, brought about by substituting the light arch for the heavy thickened pier ends, sometimes adopted in similar cases, is certainly very great.

A full and detailed description of the arch was given and its peculiar features explained.

Mr. R. Hering mentioned having seen the arch-work on 42d Street, New York, which fell during last fall, and also brick arch-work in the Brooklyn approach to the East River bridge, which showed very decided signs of giving way. He thought if the bond in the brick work of these arches had been properly broken, as described in Mr. Stauffer's paper, that they would not have failed.

Mr. Ashburner contributed interesting facts in regard to the recently completed "Cleveland Viaduct." The pivot span of this bridge is the heaviest in this country.

Mr. W. B. Ross exhibited a series of cards which he has designed for the use of engineers. They are intended to facilitate and expedite the calculation of quantities and areas. They are particularly convenient in calculating excavations and embankments. Quantities can be ascertained in less than one-third the time usually taken. The error is always a percentage of the quantity, and is about constant, being one-twentieth of one per cent.

Mr. Howard Murphy read some interesting notes upon the early waterworks of Philadelphia. It has been shown that Franklin's proposition, to supply the city with water from the Wissahickon Creek, would have given the required quantity until 1868.

CHAS. E. BILLIN, *Secretary.*

AT a meeting of the Manchester Scientific and Mechanical Society held on the 12th ult., a paper on the methods of communication between passengers and guards on railway trains was read by Mr. W. H. Bailey. The writer, after condemning the two methods at present in use, viz., the cord system and electricity, proceeded to describe a system invented by Mr. Hy. Morris, of Manchester. By this system a simple apparatus, attached to each carriage, put it in the power of any passenger to attract the attention of the guard by exploding, in rapid succession, two waterproof detonators or fog signals that were attached to the end of each carriage, and at the same time showing a red disc or semaphore at right angles to each side of the carriage, one short pull of the handle in any compartment being all that was necessary to give the signal. In the discussion which followed some slight improvements in the details of the apparatus were suggested, but generally it was highly commended as a simple and effective appliance; and very much preferable to the cord system, which had long since shown its inefficiency.

LIVERPOOL ENGINEERING SOCIETY.—This society held the first meeting of the present year on Wednesday evening, the 15th ult., at the Royal Institution. The chief business of the evening was the reading and discussion of the paper by Mr. J. S. Brodie on "The Disposal of Town Refuse." The author considered that in spite of the attention that has been paid to this all-importantant subject, and the sums of money that have been spent by various companies with greater or less results, the question as to how best to get rid of such refuse as that from dwellings, factories and slaughter-houses remains *in statu quo.* With a view to further elucidation and discussion the present paper was written, and it gives a clear and concise description of the more important schemes that have been tried for the treatment of sewage. Up to thirty years ago town refuse was either sent into the nearest watercourse or deposited on waste-land with the effect of either poisoning the stream or polluting the air. The introduction of gravitation waterworks and the w. c. system even aggravated the former evil, and a committee was appointed in 1866, which, after ten years' labor, made a report, the result of which was the Rivers Pollution Act. Natural watercourses being barred as receptacles for sewage, recourse was had to other methods of disposing of it, and Mr. Brodie considered those which had been tried as (1) Precipitation, (2) Filtration, (3) Irrigation. Under the first head Scott's lime and cement process was described and a specimen of the cement was exhibited; Whitthead's and the celebrated A.

B. C. were also described. Weare's and Bailey Denton's mode of filtration came next. Passing over irrigation as an agricultural question, the author next described the Rochdale tub system and earth closets. In conclusion, Mr. Brodie believed that unless something altogether unexpected came forward, towns would have to contrive to pay to get rid of their refuse just as they have to pay for their water supply.

IRON AND STEEL NOTES.

THE AGE OF STEEL.—Since the Admiralty approved of steel for shipbuilding, the use of iron for this purpose has rapidly diminished. The iron trade, indeed, is in a most deplorable state from this and other causes. Not only has it to cope with the prevailing depression, but it has fallen into disfavor for many of the purposes to which it was at one time exclusively applied. For torpedo launches it is altogether discarded; and there are indications that the war-ship of the future will be not an iron-clad, but a steel-clad. For the splendid vessels of the Cunard and other lines iron propellers are now a thing of the past. Only last Wednesday there was launched from Messrs. J. and G. Thomson's yard, at Glasgow, the largest Cunard steamer yet built, the Gallia, which has been fitted with four steel propeller plates, each of which weighs 65 cwt. These are the largest propeller plates ever made. They were manufactured by Messrs. John Brown & Co., Atlas Steel and Iron Works, Sheffield. Of course, these are not the first propeller plates made in steel—another Sheffield firm having been engaged in this speciality for some time. Steel is pushing iron out of the field in other departments, such as boiler-plates, bridge-plates, and even girders for house-building. For some time now the Dutch Government have employed nothing but steel in the construction of their bridges; while boiler-makers are rarely calling for iron-plates; and the difficulty of meeting Belgian competition in iron girders is being obviated by the use of steel, the carrying power of which is greatly superior to the ordinary form of girders. Altogether, the iron trade, which was at one time called "the backbone of England's commercial supremacy," is simply becoming a servant to steel, which is swiftly sweeping the ruder metal out of many markets where King Iron once reigned supreme.

NEW LIGHT ON STEEL-MAKING.—It would seem that the presence of more than one or two-tenths per cent. of phosphorus in pig-iron is no longer to be considered, as heretofore, an insuperable obstacle to its conversion into ingot steel. It was established by the Terre Noire Company, some two years ago, that as much as 0.32 per cent. of phosphorus can be tolerated in very mild steel, and, as is well known, large quantities of Martin steel made from old iron rails and pure pig have, by the aid of ferro-manganese, been manufactured on this principle. The difference between the cost of changing old iron rails, and that of using pure materials, is, however, in most localities not sufficient to cover the extra expense of using ferro-manganese. It remained, however, an axiom with steel makers, that no removal of phosphorus could be hoped for in any direct steel process, till it was announced from the Blaenavon Ironworks that there were means by which phosphorus could be removed with certainty and economy, and that intensity of temperature was no obstacle to its removal. In confirmation of the Blaenavon experiments, we learn that very important results have been obtained in Belgium with M. Ponsard's forno-convertisseur lined with one of the Blaenavon basic preparations. The maintenance of the necessary highly basic slag was effected by the addition of lime and a certain amount of ore, as prescribed by Mr. Thomas, the patentee of the process, who assisted at the operations. In the first cast of four tons, notwithstanding that the operations were conducted under very unfavorable circumstances, an analysis of the steel showed that 90 per cent. of the phosphorus contained in the pig had been removed. An examination of samples taken at intervals shows a progressive decrease of phosphorus in the bath and its transference to the slag; the amount of silica in the latter being kept at about 22 per cent. A somewhat more basic slag is, however, generally preferred. The second cast gave very similar results. As the Ponsard apparatus is able to deal with pig very low in silicon there appears to be now no class of pig which may not be considered as available for the manufacture of steel. The only impurity which is not removed almost completely is sulphur, though this is eliminated to a considerable extent; fortunately, however, sulphur is readily removed in the blast-furnace. We understand it is now in contemplation to regularly work the Ponsard converter in combination with the new basic process on the highly phosphoretic pig of Belgium and Germany. This will give an economy of from 30 to 50 francs a ton over the use of Bessemer pig, and give a fresh life to the drooping fortunes of the manufacturers.

RAILWAY NOTES.

AT the meeting of the North-Eastern Railway Company, the chairman stated that he thought they had "broken the neck" of the signaling difficulty. They had spent in the half year on new works about £14,000, but still the cost of maintaining the signaling of the line was something like £1,000 less than a year ago, and they had now interlocked the line to nearly 2,750 places in four years and a half. In the permanent way they were now deriving the benefit of the steel rails, which were such a burden to them for a long time, and some idea of what had been in this direction may be gleaned from the fact that the company has laid something like 1,600 miles of single line, or 800 of double line wholly with steel rails, including nearly the whole extent of the main lines. The chairman added that "though he hoped they would continue to feel the advantage of steel rails, he was afraid of some process coming into play which would have a similar effect on steel as the hot blast had on the manufacture of iron, since which no good iron has been made in the country." It is curious that we have done so well with bad iron for so many years.

RAILWAY WORK IN JAPAN.—At the meeting on Tuesday, the 10th of December, at the Institution of Civil Engineers, Mr. Bateman, President, in the chair, a paper was read on "Railway Work in Japan," by Mr. W. Furniss Potter, M. Inst. C. E.

The author stated that there were, at present, 66½ miles of railway in Japan, 142¾ miles laid out, with working plans, sections, and estimates completed, and 455 miles projected, the general route only having been examined and decided upon. The earthworks of the existing lines had been made for a double way, and the bridges for a single way. The permanent way was of double-headed 60 lb. rails on the Yeddo-Yokohama and Kobe-Osaka lines; but on the Osaka-Kioto line, 60 lb. flat-bottomed rails on cross sleepers were used. The superstructure of the smaller bridges was originally of timber, but had been renewed with iron. The larger bridges were all of the Warren girder type, and as a rule of 100 ft. spans. The foundations were on brick wells 12 ft. in diameter, and on an average about 60 ft. deep. Native examples of engineering were chiefly remarkable for their temporary character. The usual foundation for the largest buildings was only a few stones on the surface of the ground. The natives were very clever in making artesian borings for water, and a detailed description of the *modus operandi* was given. The workmen were extremely intelligent and industrious, especially the carpenters, who were by far the most numerous and skillful. The wages of first-class carpenters were 1*s.* 8*d.* per day; of blacksmiths, 1*s.* 6*d.*; of bricklayers and masons, 1*s.* 5*d.*, and of coolies, 11*d.* Materials found in the country for construction were not very good, except timber, which was abundant. No limestone possessing hydraulic properties had been found. It was impossible to furnish any reliable information as to the cost of the works, as the Japanese officials avoided giving particulars on this point to the foreign staff. The chief engineering difficulty in Japan was the treatment of the water-shed. The beds of the rivers were nearly all higher than the surrounding country, varying from a few feet to 40 ft., or more. In some instances the railway had been taken under the rivers by tunneling, and an example of this was given. As a rule, however, the rivers were bridged over, and approached by steep gradients, and high embankments. The flood-waters were confined in the rivers by huge banks which were gradually built up by the natives, as the beds of the river became silted up, and were frequently formidable works. The general character of the country was a series of highly-cultivated and well-watered plains, bounded by ranges of hills of the metamorphic formation. Where these hills had to be crossed there would be some heavy works. These features were described in detail. The traffic on the railways already constructed was considerable, and it was estimated that on future railways the passenger traffic alone would pay a dividend of seven per cent. Not much had been done in goods traffic, as the existing lines were in competition with the water communications. In the future development of railway work in Japan, two essential points were necessary, greater economy of construction, and the introduction of English capital and enterprise. These could be obtained if the principle of surface lines were adopted, and the natural jealousy of the government, of foreign interference, were abolished.

ENGINEERING STRUCTURES.

THE GREAT HUNGARIAN TUNNEL.—On the 21st October the great Josef adit at Schemnitz in Hungary was opened. The works have been carried on since 1872, the Hungarian Government granting £10,000 a year toward them. The adit is over ten miles long, being some 50 yards longer than Mont Cenis Tunnel. The total cost of the undertaking was £459,900; it was carried out entirely by Hungarian enterprise, and partly with Hungarian machinery.

RECORD OF A FLOATING DERRICK.—The 100-ton floating derrick City of New York, designed by Engineer Isaac Newton, formerly the principal assistant of Gen. McClellan, has just been taken up. This structure has a remarkable record. It was launched over seven years ago, and has not been raised from the water since. During this period it has been used almost daily raising weights of 60 or 70 tons. At one portion of the river wall in 1875 it transported and laid 1,780 cubic yards (3,560 tons) of Beton blocks, laying this great quantity of masonry in less than 18 days in 14 feet of water. Before it was launched, its computed weight was 1,142,518 pounds; and its actual displacement on launching was found to be 1,142,644 pounds, showing a remarkably close calculation.

The derrick has, on several occasions, lifted considerably over 100 tons. On one occasion it raised a tug-boat, which had filled and sunk, from the bottom of the river and placed it on the bulkhead. Its estimated weight was 130 tons.

It lately launched Ericsson's torpedo boat Destroyer, by raising it from the ground and placing it in the water. Notwithstanding the severe constant use to which the machine has been subjected no accident has occurred, and the repairs have consisted chiefly of two renewals of the wire rope of the main fall.

ORDNANCE AND NAVAL.

IMPROVEMENTS IN STEEL ARMOR PLATES.—Certain improvements in the manufacture of steel armor plates have just been patented by Mr. Thomas Hampton, of the Phœnix Bessemer Steel Works, Sheffield.

The inventor informs us that steel plates produced under this patent will not be more costly in manufacture than the ordinary wrought-iron plates; that his method will necessitate no alteration in the existing plant or machinery, while puddling furnaces are entirely dispensed with. A large armor plate will shortly be forwarded to the Admiralty to undergo the usual tests with shot and shell. The smaller plates heretofore experimented upon have proved so satisfactory that the result of the approaching trial is awaited with confidence.

The invention consists in the combination of two distinct processes; the first being the method of building up or construction of the plate for the purpose of obtaining a quality of steel possessing great toughness, the second process consisting in case-hardening or recarbonizing the surface or part of the surface of plates so constructed.

Steel is employed which may be made by either of the processes known as the Bessemer, Siemens, Siemens-Martin, or crucible, and the ingots so made may be either rolled direct into suitable slabs, plates, or sheets, or they may be first hammered or squeezed, and afterwards rolled. The slabs, etc., so reduced, are piled in any desired number, and placed in a reberberatory or other heating furnace, and brought to a proper heat for welding together, which may be effected in the usual manner by rolling, hammering, squeezing, or other equivalents. If desired, such plates may be piled, heated and welded together, until the requisite thickness or strength is attained. The plate being brought to the desired dimensions, is now submitted to a process, or processes, of case-hardening, or recarbonizing, on either a portion or the whole of its surface, as may be required, for the purpose of imparting to it a degree of hardness which shall tend to break up and destroy any projectile striking it at the moment of impact as well as to increase its resisting power to penetration by shot or other missile.

In order to further increase the density of the steel, the plates, after being recarbonized, may be again submitted to the action of rolling, hammering, or squeezing.

A CONTRACT has been concluded by M. Sibiriakoff, of Irkutsk, in Siberia, for the building of a steamer of 350 tons burden for the purpose of going to the assistance of the Vega. It is expected that the steamer will be ready soon enough to start fully equipped with provisions in time to reach Behring's Straits by way of the Suez Canal next August, in order to assist Professor Nordenskjold and his companions. The vessel will afterwards trade to the Lena, and, if possible, even to the Yenisei. The vessel has been designed with a special view to her future service. Inside she is almost like an ordinary iron vessel covered with wood. The propeller and rudder are so constructed that they can be lifted out of the water.

BOOK NOTICES.

THE STRENGTH OF MATERIALS. By WM. KENT, M. E. Van Nostrand's Science Series No. 41. New York : D. Van Nostrand. Price 50 cts.

The author of this little treatise sets forth in a very clear manner the present state of our knowledge on the strength of such materials as are used in engineering constructions. He especially insists, and proves by quotations from accepted authorities, that our present tables are inexact and the formulas conflicting.

A thorough acquaintance with the best literature of the subject, and a complete familiarity with the methods and results of tests, are evident throughout the book.

The aim of the essay is to urge engineers and constructors to insist upon a thorough knowledge of the strength of all materials employed in important structures, and to obtain this knowledge by aid of tests more carefully made than those upon which many of our present tables are based.

PRACTICAL THEORY OF VOUSSOIR ARCHES APPLIED TO STONE BRIDGES, TUNNELS, DOMES, AND GROINED ARCHES. By WM. CAIN, C. E. Van Nostrand's Science Series No. 42. New York : D. Van Nostrand. Price 50 cts.

Scarcely any branch of practical engineering offers so many difficulties to the young engineer as the construction of the voussoir arch. The theoretical examples of the books fail to meet in his mind the practical requirements. So he often follows precedent, and works from copy or by rule of thumb, and completes the work in ignorance whether such failure as there may be lies on the side of clumsiness or insecurity, with a lingering fear that, taken as a whole, the structure possesses both of these qualities.

Prof. Cain has previously given us so complete a discussion of the principles involved in the practical construction of arches (Science Series No. 12) that experienced engineers have employed it as a guide in important works. Taken with the present work, a treatise is afforded whose completensss in a practical point of view is scarcely equaled by anything else in the English language.

TEXT-BOOK ON THE STEAM ENGINE. By T. M. GOODEVE, M. A. New York : D. Van Nostrand. Price $2.00.

As a text book this is the most satisfactory treatise we have seen.

It is full without being voluminous, and accurately scientific without the forbidding array of analytical formulas that so often abound in similar books for students.

The plan of the work is simple and comprehensive. First is presented a historical sketch of the steam engine as it was known before Watts' time. Then an elucidation of the principles of thermodynamics and their application in the use of steam.

A chapter on mechanism follows and then a discussion of the principles of expansion—the theory of valve motion and the Indicator. Boilers and the consumption of fuel receive their share of attention ; compound engines, the injector, link work, and miscellaneous details close the work.

Examination questions are furnished as a guide to both teacher and student.

The book is abundantly illustrated.

PROF. DUBOIS'S GRAPHICAL STATICS.

In our issue for November appeared a portion of a review of the above American book, translated from Wochenschrift des Vereins deutscher Ingenieure, in which the critic charged that Prof. Dubois had used the work of Prof. Weyrauch without proper acknowledgement.

To this Prof. Dubois replied at considerable length in the December issue of this magazine, strongly asserting that there was no

proper ground for the charge, at the same time complaining that the translation was of a part of the original article only.

In the January magazine to satisfy the demands of our contributor of the translation, urged in consequence of the complaint of Prof. Dubois, we gave the entire article.

We are now offered plenty of material upon both sides of the controversy, which, at present, we decline to publish.

It is due, however, to Prof. Dubois to say that a reply from Prof. Weyrauch, has appeared in the journal which contained the original criticism, in which Prof. W. says he has not received the impression that the extracts from his work were so imperfectly acknowledged that the American reader would be misled in regard to the extent of the obligation to the German original, and expresses satisfaction with the efforts of Prof. Dubois to bring about "mutual good understanding and relations between German and American students."

EXAMPLES OF STEAM, AIR, AND GAS ENGINES OF THE MOST RECENT APPROVED TYPES, as employed in Mines, Factories, Steam Navigation, Railways and Agriculture, practically described; with an account of all the principal projects for the production of Motive Power from Heat, which have been propounded in Different Times and Countries. By JOHN BOURNE, C. E. London: Longmans. For sale by D. Van Nostrand. Price $30.00.

The publication of the present work, which was begun in 1868, was continued in monthly parts till 1870, when the author intimated that, as he was desirous of embodying in it some new information, which he could not at that moment communicate to the public, he proposed to suspend the publication for a short time to enable this addition to be made, in order that the work might be rendered more complete than would otherwise be possible. The author was at that time engaged in a series of elaborate experiments to determine in what way coal-dust could be best utilized in the generation of motive power.

In the present work the author has recapitulated the best proportions for Compound Engines and of the boilers suitable for working them. He has also recapitulated the leading features of all the principle Furnaces, Engines, or other expedients for the production of power from heat, and in some cases, as in the Air Engine of Stirling and the Caloric Engine of Ericsson, the author has described the structure in detail. Examples have been given of modern steam engines and boilers of every class; and in the appendix a variety of useful information, which could not be conveniently embodied in the text, has been introduced. While, therefore, the engineer will here find examples of the most modern and approved forms of engine construction, he will also find such a recapitulation of ingenious but unfruitful projects as may aid the progress of invention, prevent future waste of effort, and at all events satisfy an intelligent curiosity. The author believes that he has omitted to notice no project of the least importance for the production of motive power, and in most cases he has given his opinion as to the character of its pretensions.

MISCELLANEOUS.

PROF. STANLEY JEVONS still believes in his theory of the sunspots and commercial crises, and he requests our "so-called practical men to give up the idea that theory is all nonsense." But where is this particular theory, and what is it, and when we have it shall we be able to prevent commercial crises without preventing the spots on the sun's face?

CONSIDERING the unfavorable economical results hitherto obtained from steam turbines, Herr Muller, of Cologne, has lately patented a machine in which, instead of one turbine wheel several are placed together in a common case divided into a corresponding number of chambers of increasing size. The steam (or water under pressure) entering at the smaller end, finds an ever increasing cross-section of passage, and gives impulse to the successively larger turbine wheels on the common axis.

GILDING ON GLASS.—A new process by M. Dodon, is thus given by the *Moniteur de la Céramique:* Gold, chemically pure, is dissolved in aqua regia (1 part nitric and 3 parts hydrochloric acid). The solution effected, the excess of acids is evaporated on a water-bath till crystallization of the chloride of gold takes place; it is then taken off and diluted with distilled water of such quantity as to make a solution containing 1 gram of gold to 200 cubic centimeters of liquid; a solution of caustic soda is then added until the liquid exhibits an alkaline reaction. The solution of gold is now ready for reduction. As a reducing agent, an alcoholic solution of common illuminating gas is used. This is prepared by simply attaching a rubber tube to a gas-jet and passing the current of gas for about an hour through a quart of alcohol. This liquid (which should be kept in a closed vessel) is added in quantities of from two to three cubic centimeters to 200 cubic centimeters of the alkaline solution of gold before mentioned; the liquid soon begins to turn to a dark green color, and at length produces the metallic layer of gold of known reflecting power.

As an improvement on the process, as well as for convenience in executing it, there may be added to the alcoholic solution of gas an equal quantity of glycerine (28° to 30° Baumé) previously diluted with its own volume of distilled water.

If the gold employed is an alloy, the foreign metals must, in all cases, be first removed; and especially the least traces of silver, because the very smallest quantity of this metal totally prevents the regular and uniform deposition of the gold.

The bath thus once prepared, it is proposed as a method of gilding mirrors, and also for all the articles of various branches of industry, where this process of gilding could be used with success and to advantage, such, for instance, as boxes, necklace beads, candlesticks, glass ornaments, frames of table mirrors, cups, saucers, spoons, lanterns, and reflectors, and for objects generally in glass or crystal that are capable of being completely gilded.